VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education

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STUDENT NUMBER | S | O | L | U | T | I | O | N | S | Letter

# FURTHER MATHEMATICS

ANSWERS - ORANGE. **Written examination 2** PWE

EXAMINAR COMMENTS - BLUE **Wednesday 14 September 2016** 2016

THINKING - GREEN **Reading Time** 3:00 to 3:15 (15 minutes)

**Writing Time** 3:15 to 4:45 (1 hour 30 minutes)

## QUESTION AND ANSWER BOOK

| | | | |
|---|---|---|---|
| Section A – Core | *Number of questions* | *Number of questions to be answered* | *Number of marks* |
| | 9 | 9 | 36 |
| Section B – Modules | *Number of modules* | *Number of modules to be answered* | *Number of marks* |
| | 4 | 2 | 24 |
| | | | Total 60 |

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

**Materials supplied**

- Question and answer book of 30 pages.
- Formula sheet.
- Working space is provided throughout the book.

**Instructions**

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

## SECTION A – Core

### Instructions for Section A

Answer **all** questions in the spaces provided. Write using blue or black pen.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, $\pi$, surds or fractions.

In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

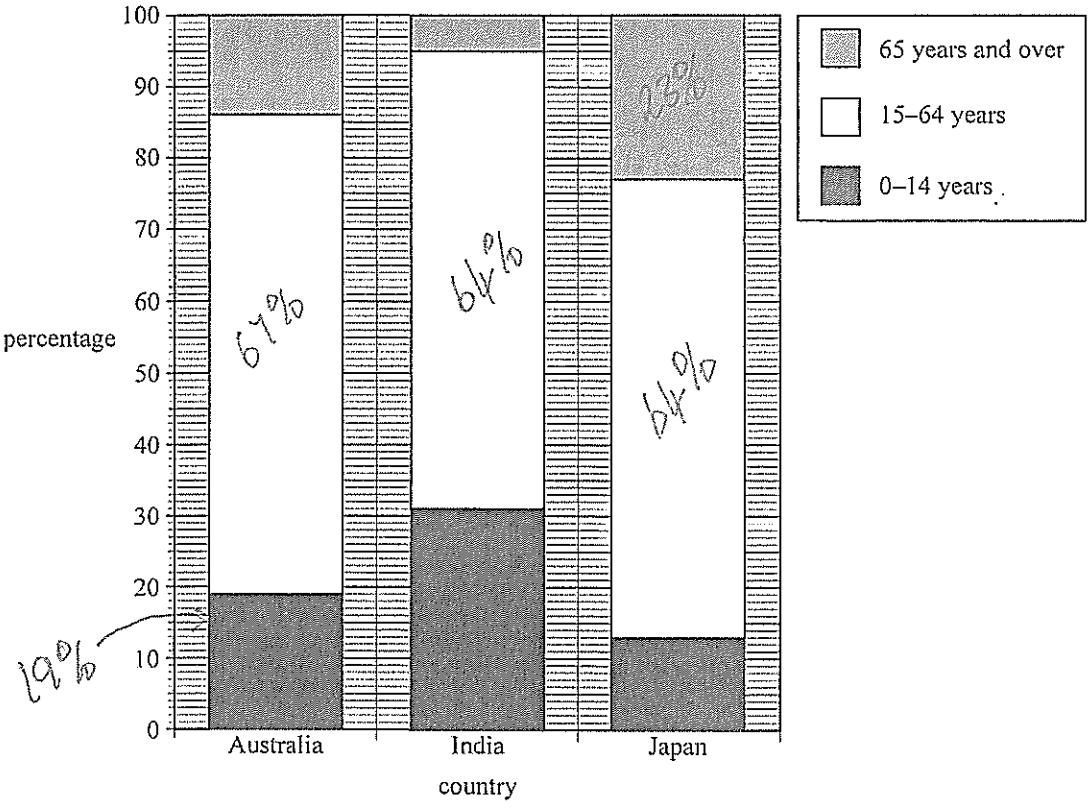
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

### Data analysis

2016 Q1

**Question 1** (3 marks)

The segmented bar chart below shows the age distribution of people in three countries, Australia, India and Japan, for the year 2010.




Source: Australian Bureau of Statistics, 3201.0 – *Population by Age and Sex, Australian States and Territories*, June 2010

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a. Write down the percentage of people in Australia who were aged 0–14 years in 2010. 1 mark

19% (read from bar chart).

b. In 2010, the population of Japan was 128 000 000.

How many people in Japan were aged 65 years and over in 2010? 1 mark

29,440,000 (23% of 128,000,000)

($\frac{23}{100} \times 128{,}000{,}000 = 29{,}440{,}000$)

c. From the graph on page 2, it appears that there is no association between the percentage of people in the 15–64 age group and the country in which they live.

Explain why, quoting appropriate percentages to support your explanation. 1 mark

All three countries have approximately the same percentage 67%, 64% and 64% respectively.

Working Out.

Australia 86% − 19% = 67%

India 95% − 31% = 64%

Japan 77% − 13% = 64%

Examiners Report

1a. 93% got 1 mark.

1b. 56% got 1 mark. Answer must be written in full (not 2.944E7, calculator form). The number in Full or Scientific Notation 29440000 or $2.944 \times 10^7$ is OK.

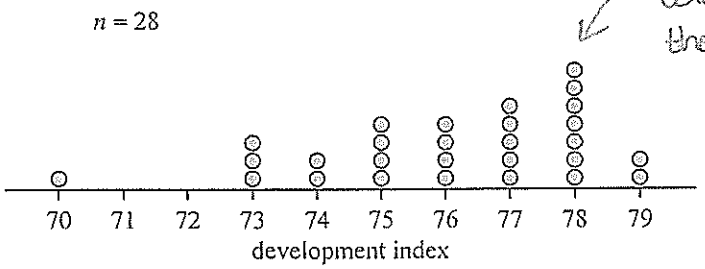
1c. 41% correct (1 mark)
However, lots of students contradicted the statement and said there "was an association because...."

**Question 2** (3 marks) Q2 2013

The development index for a country is a whole number between 0 and 100.

The dot plot below displays the values of the development indices for 28 countries.

mode is the value that occurs the most.




**a.** Using the information in the dot plot, determine each of the following. 1 mark

The mode 78

The range 9

min value = 70
max value = 79
79 - 70 = 9

**b.** Write down an appropriate calculation and use it to explain why the country with a development index of 70 is an outlier for this group of countries. 2 marks

Thinking. For outlier calculation we need $Q_1$ and $Q_3$ to find IQR. Use CAS or manually.

$Q_1 = 75$, $Q_3 = 78 \Rightarrow IQR = 78 - 75 = 3$.

Lower fence/boundary $= Q_1 - 1.5 \times IQR$

$= 75 - (1.5 \times 3)$

$= 75 - (4.5)$

$= 70.5$.

Therefore 70 is an outlier because it is less than the lower fence/boundary of 70.5.

Examiners Comments: For the 2 marks, Need
1. an answer for lower fence.
2. a comparison of this to the value 70
3. ~~calculation of lower fence.~~
4. statement of that "it is an outlier"

mark.

Marks for Q2 (2013)

| | |
|---|---|
| 8% | 0 |
| 28% | 1 |
| 12% | 2 |
| 52% | 3 |

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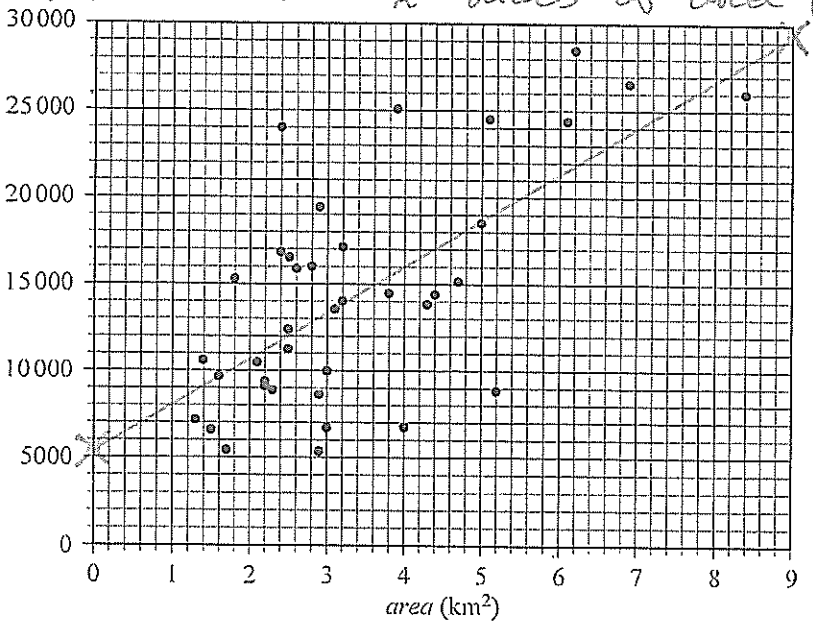
**Question 3** (6 marks) Q2 [2014]

The scatterplot below shows the *population* and *area* (in square kilometres) of a sample of inner suburbs of a large city.

(b) To draw, least squares regression line, use the formula and 2 values of area (min and max are recommended).




area = 0
pop = 5330

a = 9.
P = 29450

The equation of the least squares regression line for the data in the scatterplot is

$$population = 5330 + 2680 \times area$$

a. Write down the response variable. 1 mark

Population (86% correct, in 2014)

b. Draw the least squares regression line on the **scatterplot above**. 1 mark

(*Answer on the scatterplot above.*) see above.

(36% got 1 mark, 64% got 0 marks)

c. Interpret the slope of this least squares regression line in terms of the variables *area* and *population*. 2 marks

On average, population increases by 2680 people for each additional square kilometre of area.

Examiners Report: (55% - 0 marks, 8% - 1 mark, 37% - 2 marks)

Must state the value of the gradient and in this case the correct unit of area.

For Part(b) You must clearly show the use of the regression formula. Also, your data points need to be accurate as possible!

**d.** Wiston is an inner suburb. It has an area of 4 km$^2$ and a population of 6690.
The correlation coefficient, $r$, is equal to 0.668

**i.** Calculate the residual when the least squares regression line is used to predict the population of Wiston from its area. 1 mark

The area = 4 km$^2$ Pop = 5330 + 2680 × 4 = 16050

Residual = Population − Population(predicted)

= 6690 − 16050

= −9360 people.

**ii.** What percentage of the variation in the population of the suburbs is explained by the variation in area?
Round your answer to one decimal place. 1 mark

Variation = $r^2$, $r \doteq 0.668$

$r^2 = 0.446224$

$= 44.6\%$

Examiner Report.
Part d: 48% 0 marks
21% 1 marks
31% 2 marks.

d) i) Must have negative sign.

ii) Must be to 1 decimal place.
Must not be 45%
44.6224% is marked correct
(marker will round this).

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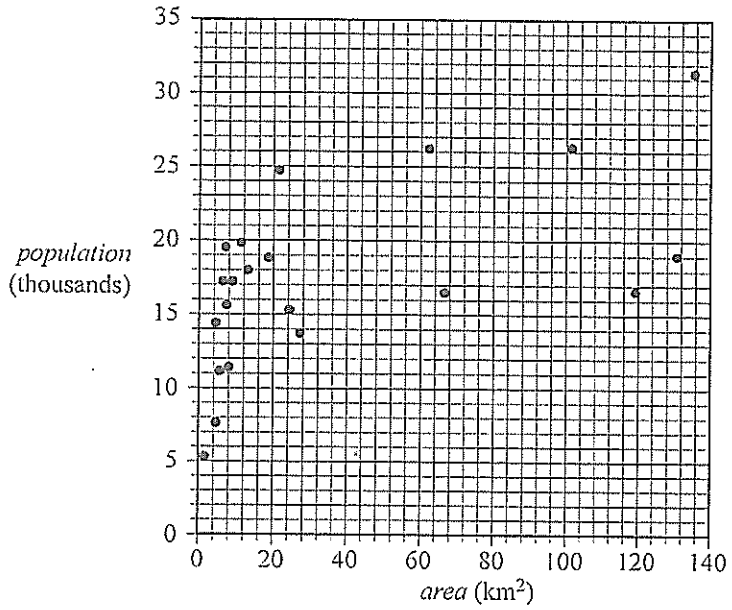
**Question 4** (3 marks) Q3 2014

The scatterplot and table below show the *population*, in thousands, and the *area*, in square kilometres, for a sample of 21 outer suburbs of the same city.




| *Area* (km²) | *Population* (thousands) |
|---|---|
| 1.6 | 5.2 |
| 4.4 | 14.3 |
| 4.6 | 7.5 |
| 5.6 | 11.0 |
| 6.3 | 17.1 |
| 7.0 | 19.4 |
| 7.3 | 15.5 |
| 8.0 | 11.3 |
| 8.8 | 17.1 |
| 11.1 | 19.7 |
| 13.0 | 17.9 |
| 18.5 | 18.7 |
| 21.3 | 24.6 |
| 24.2 | 15.2 |
| 27.0 | 13.6 |
| 62.1 | 26.1 |
| 66.5 | 16.4 |
| 101.4 | 26.2 |
| 119.2 | 16.5 |
| 130.7 | 18.9 |
| 135.4 | 31.3 |

In the outer suburbs, the relationship between *population* and *area* is non-linear.
A **log** transformation can be applied to the variable *area* to linearise the scatterplot.

50% correct in 2014.

a. Apply the **log** transformation to the data and determine the equation of the least squares regression line that allows the population of an outer suburb to be predicted from the logarithm of its area.

Write the slope and intercept of this least squares regression line in the boxes provided below.

Round your answers to two significant figures.

Note in 2014 this was 1 decimal place NOW it is 2 sig. figs.

2 marks

*population* = [7.7] + [7.7] log (*area*)

b. Use the equation of the least squares regression line in **part a.** to predict the population of an outer suburb with an area of 90 km².

Round your answer to the nearest one thousand people.

1 mark

23,000 people.

30% correct in 2014 Most forgot to use $\log_{10}(90)$

Population = $7.7 + 7.7 \times \log_{10}(90)$

= 22.7477 thousand

or 23000 people

look at graph!! 23000 makes sense 700000 doesn't!

Not using $\log_{10}(90)$ gave an ans of 700K

**Question 5** (4 marks) Q4 [2014] modified.

There is an association between the variables *population density*, in people per square kilometre, and *area*, in square kilometres, of 38 inner suburbs of the same city.

For this association, $r^2 = 0.141$

**a.** Write down the value of the correlation coefficient for this association between the variables *population density* and *area*.

Round your answer to three decimal places. 1 mark

$r = \sqrt{r^2} = \sqrt{0.141} = 0.375$

**b.** The mean and standard deviation of the variables *population density* and *area* for these 38 inner suburbs are shown in the table below.

| | *Population density* (people per km$^2$) | *Area* (km$^2$) |
|---|---|---|
| **Mean** | 4370 | 3.4 |
| **Standard deviation** | 1560 | 1.6 |

One of these suburbs has a population density of 3082 people per square kilometre.

**i.** Determine the standard $z$-score of this suburb's population density.

Round your answer to one decimal place. 1 mark

Standard score $z = \frac{x - \bar{x}}{s_x} = \frac{3082 - 4370}{1560} = -0.825$

$= -0.8$

**ii.** Interpret the $z$-score of this suburb's population density with reference to the mean population density. 1 mark

altered.

The suburb's population density is 0.8 of the standard deviation lower than the mean population.

**iii.** Assume the areas of these inner suburbs are approximately normally distributed.

How many of these 38 suburbs are **expected** to have an area that is two standard deviations or more above the mean?

Round your answer to the nearest whole number. 1 mark

2.5% of 38 suburbs $= \frac{2.5}{100} \times 38$

$= 0.95$

$= 1$

1 suburb.

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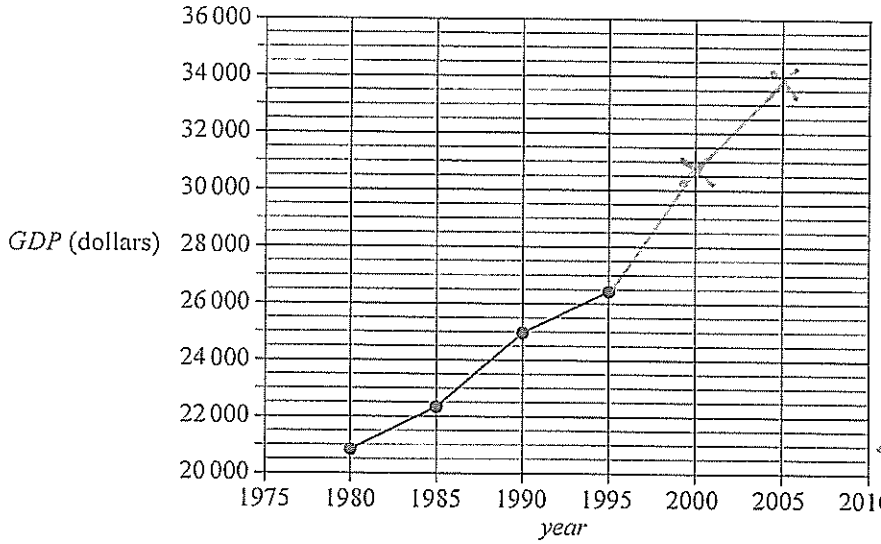
**Question 6** (5 marks) 2010 Q3

Table 1 shows the Australian gross domestic product (*GDP*) per person, in dollars, at five yearly intervals (*year*) for the period 1980 to 2005.

**Table 1**

| *Year* | 1980 | 1985 | 1990 | 1995 | 2000 | 2005 |
|---|---|---|---|---|---|---|
| *GDP* | 20 900 | 22 300 | 25 000 | 26 400 | 30 900 | 33 800 |




* must be as accurate as possible.
* Identify scale
↓
← 22 000 ← 21500
← 21 000 ← 20500
← 20 000

**a.** Complete the **time series plot above** by plotting the *GDP* for the years 2000 and 2005. 1 mark

(*Answer on the time series plot above.*)

**b.** Briefly describe the general trend in the data. 1 mark

An increasing trend

**c.** In Table 2, the variable year has been rescaled using 1980 = 0, 1985 = 5, and so on. The new variable is *time*.

**Table 2**

| *Year* | 1980 | 1985 | 1990 | 1995 | 2000 | 2005 |
|---|---|---|---|---|---|---|
| *Time* | 0 | 5 | 10 | 15 | 20 | 25 |
| *GDP* | 20 900 | 22 300 | 25 000 | 26 400 | 30 900 | 33 800 |

**i.** Use the variables *time* and *GDP* to write down the equation of the least squares regression line that can be used to predict *GDP* from *time*. Take *time* as the explanatory variable. 2 marks

GDP = 20000 + 524.4 × time

**ii.** The least squares regression line in **part c.i.** above has been used to predict the *GDP* in 2010.

Explain why this prediction is unreliable. 1 mark

2010 is outside the existing data range & is therefore an extrapolation.

OR.

The regression equation would make a prediction about the GDP in the future, which is outside the available data.

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## Recursion and financial modelling

2013 Q1 modified.

**Question 7** (4 marks)

Hugo is a professional bike rider.

The value of his bike will be depreciated over time using the flat rate method of depreciation.

The value of Hugo's bike, in dollars, after $n$ years, $V_n$, can be modelled using the recurrence relation below.

$$V_0 = 8400, \qquad V_{n+1} = V_n - 1200$$

**a.** Using the recurrence relation, write down calculations to show that the value of Hugo's bike after two years is \$6000. 1 mark

$V_1 = V_0 - 1200$, $V_0 = 8400$, $V_2 = V_1 - 1200$

$V_1 = 8400 - 1200$ $V_2 = 7200 - 1200$

$V_1 = 7200$ $V_2 = 6000$

Hence, it is worth \$6000 after 2 years.

Hugo will sell his bike when its value reduces to \$3600.

**b.** After how many years will Hugo sell his bike? 1 mark

Calculate $V_3 = 6000 - 1200 = 4800$

$V_4 = 4800 - 1200 = 3600$

Hugo will sell his bike after 4 years.

The unit cost method can also be used to depreciate the value of Hugo's bike.

A rule for the value of the bike, in dollars, after travelling $n$ kilometres is

$$V_n = 8400 - 0.25n$$

**c.** What is the depreciation of the bike per kilometre? 1 mark

(The coefficient of $n$ is 0.25) Depreciation is \$0.25 per kilometre.

After two years, the value of the bike when depreciated by the unit cost method will be the same as the value of the bike when depreciated by the flat rate method.

**d.** How many kilometres has the bike travelled after two years? 1 mark

$V_2 = 6000 = 8400 - 0.25n$, find $n$.

Using CAS $n = 9600$ km.

No, Examiners Report since new to study design.

**Question 8** (5 marks) Q2 [2013] modified.

Hugo won \$5000 in a road race. He deposited this money into a savings account.

The value of Hugo's savings after $n$ months, $S_n$, can be modelled by the recurrence relation below.

$$S_0 = 5000, \qquad S_{n+1} = 1.004\, S_n$$

**a.** What is the annual interest rate (compounding monthly) for Hugo's savings account? 1 mark

$R = 1 + \frac{r}{100}$, where $R = 1.004$

$1.004 = 1 + \frac{r}{100}$

$\Rightarrow \quad r = 0.4\%$ per month or $12 \times 0.4\% = 4.8\%$ p.a.

**b.** What would be the value of Hugo's savings after 12 months? 1 mark

$S_{12} = 5000(1.004)^n$, $n = 12$ months.

$= \$5245.35$

Using a different investment strategy, Hugo could deposit \$3000 into an account earning compound interest at the rate of 4.2% per annum, compounding monthly, and make additional payments of \$200 after every month.

Let $T_n$ be the value of Hugo's investment after $n$ months using this strategy.

The monthly interest rate for this account is 0.35%.

if compounding use Fin. Solver.

**c.** **i.** Write down a recurrence relation, in terms of $T_{n+1}$ and $T_n$, that models the value of Hugo's investment using this strategy. 1 mark

$T_{n+1} = RT_n + d$, $R = 1 + \frac{r}{100} = 1.0035$

So $T_0 = 3000$, $T_{n+1} = 1.0035T_n + 200$, $d = \$200$.

answer.

**ii.** What is the total interest Hugo would have earned after six months? 2 marks

Formula is $T_{n+1} = T_0 R^n + \frac{d(R^n - 1)}{R - 1}$

where $d$ is regular payments and $R = \left(1 + \frac{r}{100}\right)$

$T_6 = 3000(1.0035)^6 + \frac{200(1.0035^6 - 1)}{(1.0035 - 1)}$

$= 4274.10$

Interest $= T_6 - T_0 = 4274.10 - 3000$

$= \$1274.10$

easier

OR Fin. Solver.

$n = 6$

$I = 0.35 \times 12 = 4.2$.

$PV = -3000$ (-ve because deposit)

$Pmt = -200$ ( " " " )

$F.V. = ?$ $FV = 4274.10$.

$PpY = 12$, $CpY = 12$.

OR recursion

$T_1 =$

$T_2 =$

$T_3 =$ ...

⋮



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**Question 9** (3 marks)

Hugo needs to buy a new bike.

He borrowed $7500 to pay for the bike and will be charged interest at the rate of 5.76% per annum, compounding monthly.

Hugo will fully repay this loan with repayments of $430 each month.

a. How many repayments are required to fully repay this loan?

Round your answer to the nearest whole number. 1 mark

Finance Solver. n=?, I=5.76%, PV=7500, pmt=-430

PpY=12, CpY=12, FV=0, n=18.259

I think it should be 19 to fully repay

number of repayments is 18.

After the fifth repayment, Hugo increased his monthly repayment so that the loan was fully repaid with a further seven repayments (that is, 12 repayments in total).

b. i. What is the minimum value of Hugo's new monthly repayment? 1 mark

Think 12 payments in total, so after 5 payments there are 7 left. So, first find how much owing after 5. n=5, I=5.76, 7500, p=-430, FV=5511, use as new PV for n=7. Then pmt = 802.474, so monthly is $802.47

ii. What is the value of the final repayment required to ensure the loan is fully repaid after 12 repayments? 1 mark

Yes, the rounded answer.

Now if he pays $802.47 a month for 7 months whats left.

N=7, I=5.76, PV=-5511, pmt=802.47

PpY=12, CpY=12, FV=0.028=0.03 dollars or 3 cents.

So, the final payment is $802.47+0.03

=$802.50.

**END OF SECTION A**

**TURN OVER**

## SECTION B – Modules

### Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules. Write using blue or black pen.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, $\pi$, surds or fractions.

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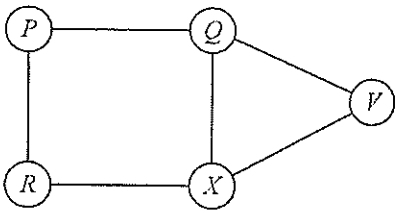
## Module 1 – Matrices 2013 Q1 modified.

**Question 1** (2 marks)

Five trout-breeding ponds, $P$, $Q$, $R$, $X$ and $V$, are connected by pipes, as shown in the diagram below.




The matrix $W$ is used to represent the information in this diagram.

$$W = \begin{array}{c} \begin{matrix} P & Q & R & X & V \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array} \begmatrix} P \\ Q \\ R \\ X \\ V \end{matrix}$$

→ = 2

In matrix $W$:

- the 1 in row 2, column 1, for example, indicates that pond $P$ is directly connected by a pipe to pond $Q$
- the 0 in row 5, column 1, for example, indicates that pond $P$ is not directly connected by a pipe to pond $V$.

**a.** In terms of the breeding ponds described, what does the sum of the elements in row 3 of matrix $W$ represent?

1 mark

Two ponds connect directly, via a pipe to pond R.

The matrix $W^2$ is shown below. To

$$W^2 = \begin{array}{c} \begin{matrix} P & Q & R & X & V \end{matrix} \\ \begin{bmatrix} 2 & 0 & 0 & 2 & 1 \\ 0 & 3 & 2 & 1 & 1 \\ 0 & 2 & 2 & 0 & 1 \\ 2 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix} \end{array} \begmatrix} P \\ Q \\ R \\ X \\ V \end{matrix}$$

From.

**b.** Matrix $W^2$ has a 2 in row 2 ($Q$), column 3 ($R$).

Explain what this number tells us about the pipe connections between $Q$ and $R$.

1 mark

That, pond R is connected to pond Q through another pond by pipes in 2 different ways.

**Question 2** (10 marks) 2013 Q2.

10000 trout eggs, 1000 baby trout and 800 adult trout are placed in a pond to establish a trout population.

In establishing this population:

- eggs ($E$) may die ($D$) or they may live and eventually become baby trout ($B$)
- baby trout ($B$) may die ($D$) or they may live and eventually become adult trout ($A$)
- adult trout ($A$) may die ($D$) or they may live for a period of time but will eventually die.

From year to year, this situation can be represented by the transition matrix $T$, where

$$T = \begin{array}{c} \\ E \\ B \\ A \\ D \end{array} \begin{array}{c} \begin{array}{cccc} E & B & A & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ \\ \\ \textit{next year} \end{array}$$

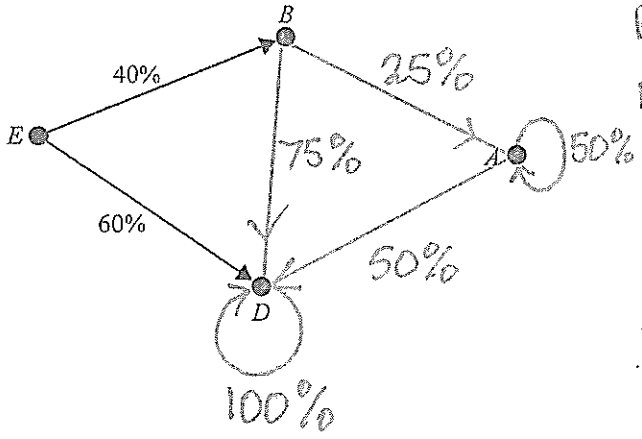
*this year*

**a.** Use the information in the transition matrix $T$ to

**i.** determine the number of eggs in this population that die in the first year 1 mark

60% of 10,000 = $\frac{60}{100} \times 10{,}000 = 6000$

**ii.** complete the transition diagram below, showing the relevant percentages. 2 marks




B → A is 25%
B → D is 75%
A → A is 50%
A → D is 50%
D → D is 100%
(Dead stay dead)

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The initial state matrix for this trout population, $S_0$, can be written as

$$S_0 = \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} \begin{matrix} E \\ B \\ A \\ D \end{matrix}$$

Let $S_n$ represent the state matrix describing the trout population after $n$ years.

**b.** Using the rule $S_{n+1} = T S_n$, determine

**i.** $S_1 = T \times S_0 = \begin{bmatrix} 0 \\ 4000 \\ 650 \\ 7150 \end{bmatrix} \begin{matrix} E \\ B \\ A \\ D \end{matrix}$ CAS. 1 mark

Note: should still equal $10000 + 1000 + 800 = 11,800$.

**ii.** the number of adult trout predicted to be in the population after four years.
Round your answer to the nearest whole number of trout. 1 mark

$S_4 = T^4 \times S_0 = \begin{bmatrix} 0 \\ 0 \\ 331 \\ 1469 \end{bmatrix}$

331 adult trout are alive after 4 years.

**c.** The transition matrix $T$ predicts that, in the long term, all of the eggs, baby trout and adult trout will die.

**i.** How many years will it take for all of the adult trout to die (that is, when the number of adult trout in the population is first predicted to be less than one)? 1 mark

Use recursion matrices, 12 years 1.29 adults
13 years. 13 years 0.64 adults

**ii.** What is the largest number of adult trout that is predicted to be in the pond in any one year? 1 mark

Using recursion $n=0$ 800 adults, $n=1$ 650 adults
Answer 2 years. $n=2$ 1325 adults, $n=3$ 663 adults.

**d.** Determine the number of eggs, baby trout and adult trout that, if added to or removed from the pond at the end of each year, will ensure that the number of eggs, baby trout and adult trout in the population remains constant from year to year. 2 marks

This is just asking: what is $S_0 - S_1$

$\begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 4000 \\ 650 \\ 7150 \end{bmatrix} = \begin{bmatrix} 10000 \\ -3000 \\ +150 \\ \end{bmatrix}$

Answer Add 10000 eggs
remove 3000 baby
add 150 adults.

check it $S_1 = T \times S_0 + \begin{bmatrix} 10000 \\ -3000 \\ +150 \\ -7150 \end{bmatrix} = S_0$

$S_2 = T \times S_1 + \quad " \quad = S_0$

The rule $S_{n+1} = T S_n$ that was used to describe the development of the trout in this pond does not take into account new eggs added to the population when the adult trout begin to breed.

To take breeding into account, assume that every year 50% of the adult trout each lay 500 eggs.

The matrix describing the population after $n$ years, $S_n$, is now given by the new rule

$$S_{n+1} = T S_n + 500 M S_n$$

where

$$T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.40 & 0 & 0 & 0 \\ 0 & 0.25 & 0.50 & 0 \\ 0.60 & 0.75 & 0.50 & 1.0 \end{bmatrix}, \quad M = \begin{bmatrix} 0 & 0 & 0.50 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } S_0 = \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix}$$

**e.** Use this new rule to determine $S_2$. 1 mark

$S_1 = TS_0 + 500 \times M \times S_0$

$S_1 = \begin{bmatrix} 200\,000 \\ 4\,000 \\ 650 \\ 7\,150 \end{bmatrix}$

$S_2 = TS_1 + 500 \times M \times S_1$

$S_2 = \begin{bmatrix} 162\,500 \\ 80\,000 \\ 1\,325 \\ 130\,475 \end{bmatrix}$ E, B, A, D.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

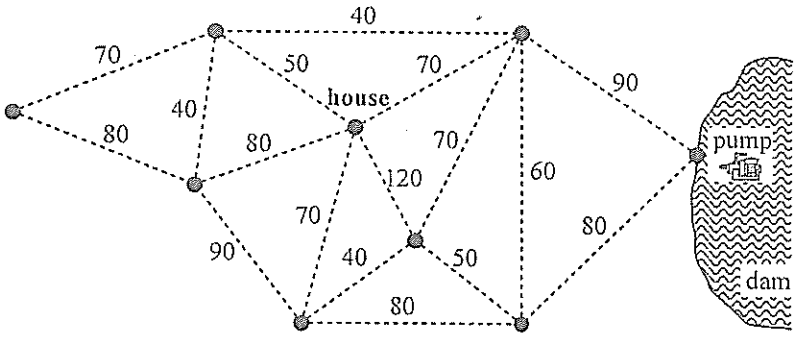
## Module 2 – Networks and decision mathematics

**Question 1** (6 marks) (2012 Q1)

Water will be pumped from a dam to eight locations on a farm.

The pump and the eight locations (including the house) are shown as vertices in the network diagram below.

The numbers on the edges joining the vertices give the shortest distances, in metres, between locations.




**a.** **i.** Determine the shortest distance between the house and the pump. 1 mark

160 m (90 + 70) (Poorly answered in 2012)

**ii.** How many vertices on the network diagram have an odd degree? 1 mark

2 (large variations 1 → 7)

**iii.** The total length of all edges in the network is 1180 m.

A journey starts and finishes at the house and travels along every edge in the network.

Determine the shortest distance travelled. 1 mark

1250 m (1180 + 70) House to top right vertex (2 odds) ⇒ 1180 + 70 to get back to house.

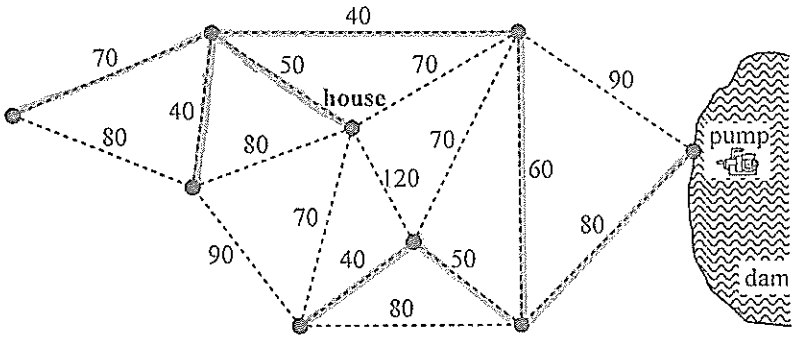
**iv.** A Hamiltonian path, beginning at the house, is determined for this network.

How many edges does this path involve? 1 mark

8 (1 less than the number of vertices, which is 9)

The total length of pipe that supplies water from the pump to the eight locations on the farm is a minimum. This minimum length of pipe is laid along some of the edges in the network.

**b.** **i.** On the diagram below, draw the minimum length of pipe that is needed to supply water to all locations on the farm. 1 mark




**ii.** What is the mathematical term that is used to describe this minimum length of pipe in **part b.i.**? 1 mark

Minimum spanning tree.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

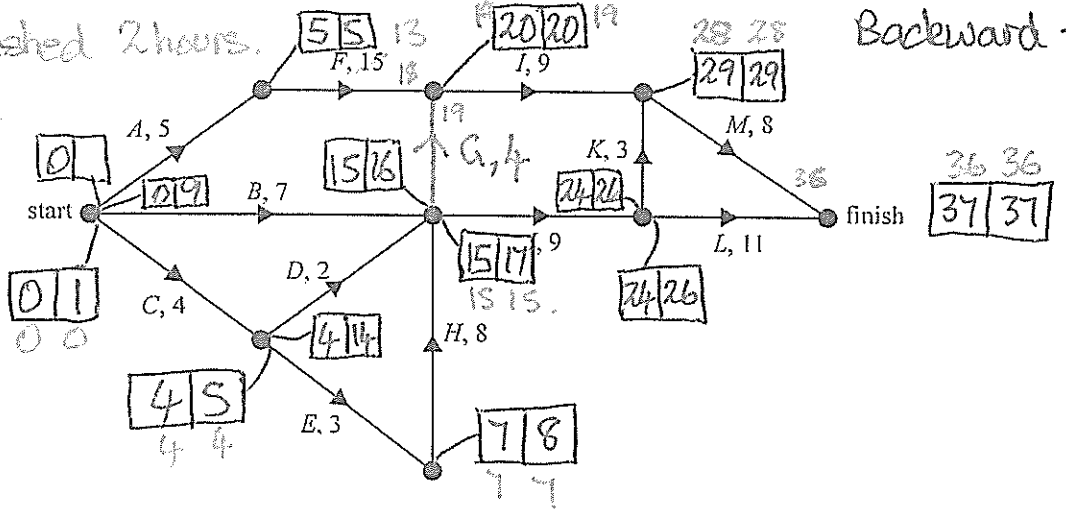
**Question 2** (6 marks) 2013 Q2

A project will be undertaken on the farm. This project involves the 13 activities shown in the table below. The duration, in hours, and predecessor(s) of each activity are also included in the table.

| Activity | Duration (hours) | Predecessor(s) |
|---|---|---|
| *A* | 5 | – |
| *B* | 7 | – |
| *C* | 4 | – |
| *D* | 2 | *C* |
| *E* | 3 | *C* |
| *F* | 15 | *A* |
| *G* | 4 | *B, D, H* |
| *H* | 8 | *E* |
| *I* | 9 | *F, G* |
| *J* | 9 | *B, D, H* |
| *K* | 3 | *J* |
| *L* | 11 | *J* |
| *M* | 8 | *I, K* |

Activity *G* is missing from the network diagram for this project, which is shown below.

F crashed 2 hours. Forward – Backward –




**a.** Complete the **network diagram above** by inserting activity *G*. 1 mark

*(Answer on the network diagram above.)*

**b.** Determine the earliest starting time of activity *H*. 1 mark

7 hrs

**c.** Given that activity *G* is not on the critical path

**i.** write down the activities that are on the critical path in the order that they are completed 1 mark

AFIM

**ii.** find the latest starting time for activity *D*. 1 mark

LST for D is 14 hours.

**d.** Consider the following statement:

'If just one of the activities in this project is crashed by one hour, then the minimum time to complete the entire project will be reduced by one hour.'

Explain the circumstances under which this statement will be true for this project. 1 mark

This will only be true if the crashed activity is one of the four activities (AFIM) on the critical path.

**e.** Assume activity *F* is crashed by two hours.

What will be the minimum completion time for the project? 1 mark

F crashed by 2 hours, new critical path (CEHGIM) Minimum completion time is now 36 hours.

DO NOT WRITE IN THIS AREA

NOT option C.

**Question 7**

The order of matrix $X$ is $3 \times 2$.

The element in row $i$ and column $j$ of matrix $X$ is $x_{ij}$ and it is determined by the rule

$$x_{ij} = i + j$$

The matrix $X$ is

A. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 2 \\ 3 & 3 \\ 4 & 4 \end{bmatrix}$

E. $\begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{bmatrix}$

$x_{ij} = i + j$

$x_{11} = 2, \quad x_{12} = 3.$

$x_{21} = 3, \quad x_{22} = 4$

$x_{31} = 4, \quad x_{32} = 5.$

$\begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{bmatrix}$

**Question 8**

A transition matrix, $T$, and a state matrix, $S_2$, are defined as follows.

$$T = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix} \qquad S_2 = \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$$

If $S_2 = TS_1$, the state matrix $S_1$ is

A. $\begin{bmatrix} 200 \\ 250 \\ 150 \end{bmatrix}$

B. $\begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$

C. $\begin{bmatrix} 300 \\ 0 \\ 300 \end{bmatrix}$

D. $\begin{bmatrix} 400 \\ 0 \\ 200 \end{bmatrix}$

E. undefined

$S_2 = TS_1$

$\Rightarrow T^{-1}S_2 = T^{-1}TS_1$

$T^{-1}S_2 = S_1$

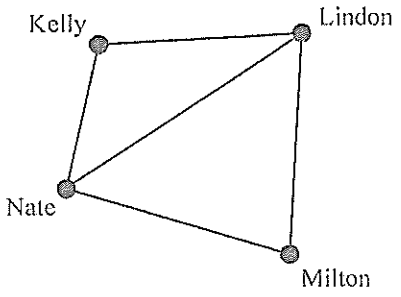
CAS $\Rightarrow S_1 = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}^{-1} \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix} = \begin{bmatrix} 400 \\ 0 \\ 200 \end{bmatrix}$

## Module 2 – Networks and decision mathematics

Before answering these questions, you must **shade** the 'Networks and decision mathematics' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

### Question 1

The graph below shows the roads connecting four towns: Kelly, Lindon, Milton and Nate.




A bus starts at Kelly, travels through Nate and Lindon, then stops when it reaches Milton.

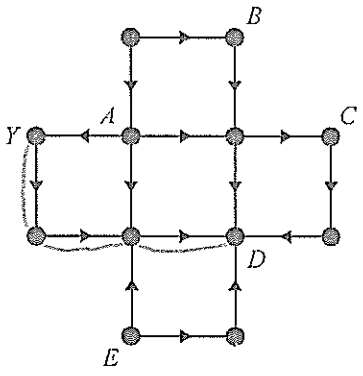
The mathematical term for this route is

**A.** a loop.
**B.** an Eulerian trail.
**C.** an Eulerian circuit.
**D.** a Hamiltonian path.
**E.** a Hamiltonian cycle.

Through each vertex!

67% correct in 2014

### Question 2




88% correct in 2014

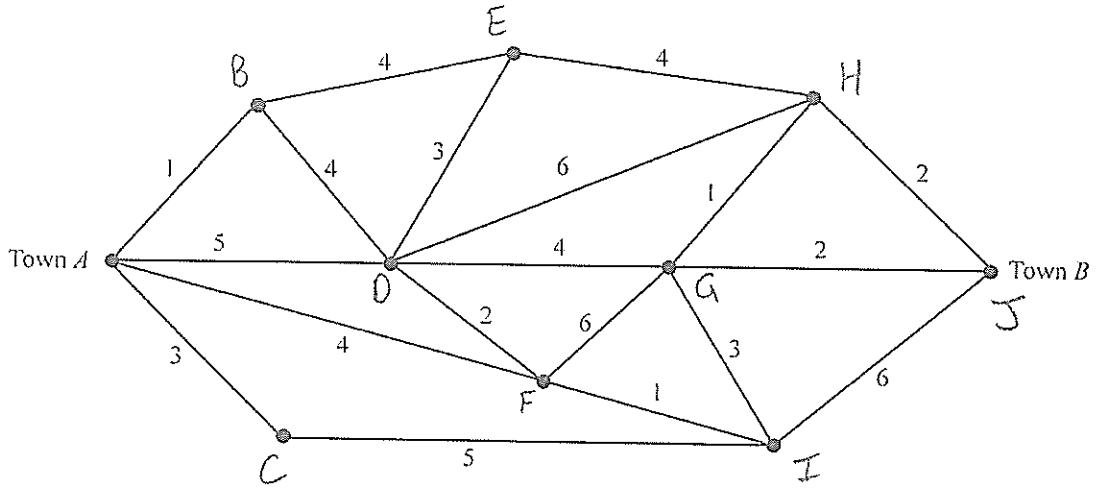
In the directed graph above, the only vertex with a label that can be reached from vertex $Y$ is

**A.** vertex $A$.
**B.** vertex $B$.
**C.** vertex $C$.
**D.** vertex $D$.
**E.** vertex $E$.

Follow arrows.

**Question 3**

The following network shows the distances, in kilometres, along a series of roads that connect Town *A* to Town *B*.



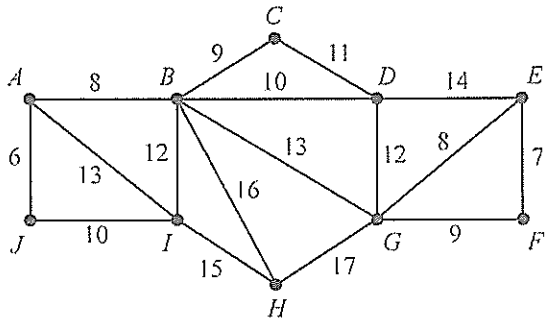

Using Dijkstra's algorithm, or otherwise, the shortest distance, in kilometres, from Town *A* to Town *B* is

A. 9
B. 10
C. 11
D. 12
E. 13

A - F - I - G - J.

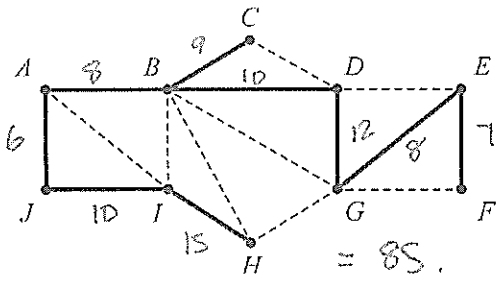
| | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|
| A | 1 | 3 | 5 | × | 4 | × | × | × | × |
| B | 1 | 3 | 5 | 5 | 4 | × | × | ✓ | × |
| C | 1 | 3 | 5 | 5 | 4 | × | × | 8 | × |
| F | 1 | 3 | 5 | 5 | 4 | 10 | × | 5 | × |
| I | 1 | 3 | 5 | 5 | 4 | 8 | × | 5 | 11 |
| D | 1 | 3 | 5 | 5 | 4 | 8 | 11 | 5 | 11 |
| E | 1 | 3 | 5 | 5 | 4 | 8 | 9 | 5 | 11 |
| G | 1 | 3 | 5 | 5 | 4 | 8 | 9 | 5 | 10 |
| H | | | | | | | | | |

**Question 4**

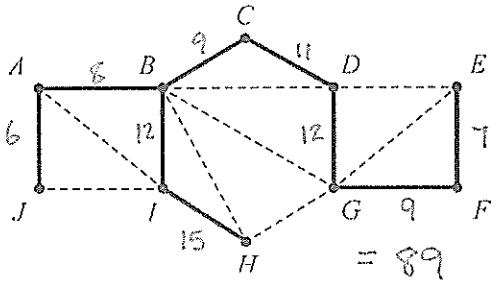



Which one of the following is the minimal spanning tree for the weighted graph shown above?

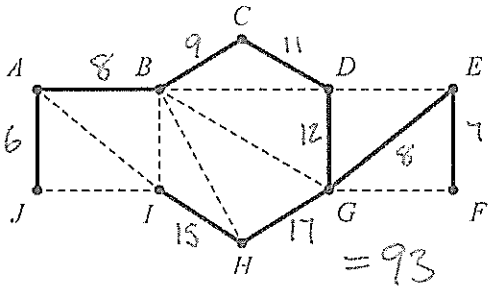
A.



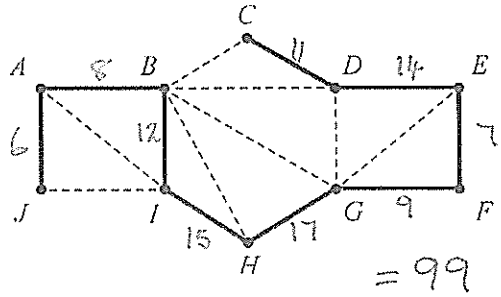

B.



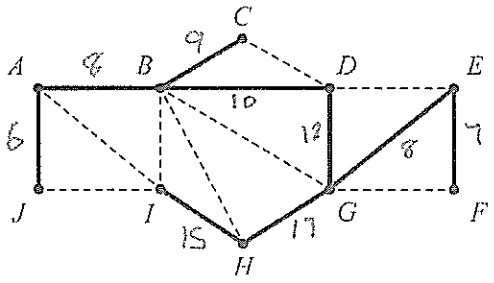

C.




D.



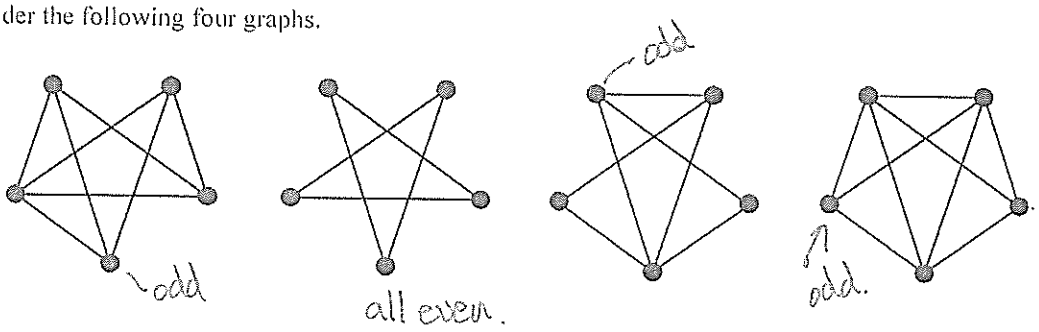

E.




84% correct in 2014

*Use the following information to answer Questions 5 and 6.*

Consider the following four graphs.




**Question 5**

How many of the four graphs above have an Eulerian circuit?

A. 0
B. 1
C. 2
D. 3
E. 4

for Eulerian Circuit all degrees EVEN

69% correct in 2014

**Question 6**

How many of the four graphs above are planar?

A. 0
B. 1
C. 2
D. 3
E. 4

To be NON-planar, All 5 vertices graphs would need to be complete - all vertices connected to each other. All 4 are not complete.

7% correct
58% incorrect option A

**Question 7**

Which one of the following statements about critical paths is **true**?

A. There can be only one critical path in a project.
B. A critical path always includes at least two activities.
C. A critical path will always include the activity that takes the longest time to complete.
D. Reducing the time of any activity on a critical path for a project will always reduce the minimum completion time for the project.
E. If there are no other changes, increasing the time of any activity on a critical path will always increase the completion time of a project.

(A) Not true as two paths could take the same time to complete and be the longest.

(B) One activity could be the longest - Not true.

(C) May or May not have longest activity - Not true.

(D) Not true if there are 2 critical paths.

(E) True

40% correct in 2014

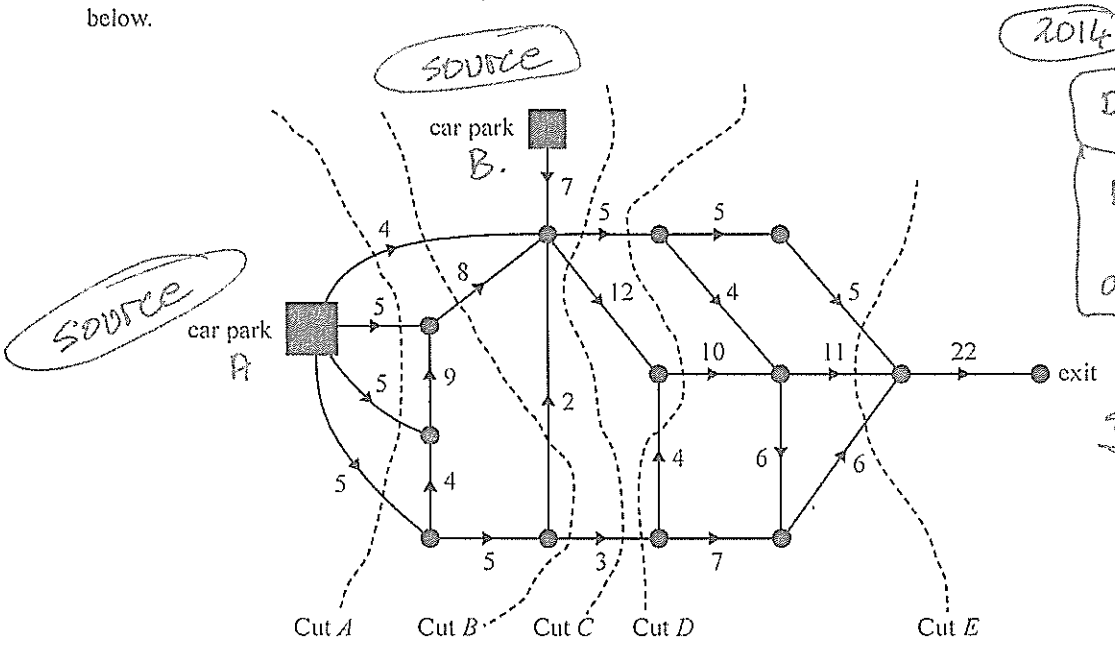
**Question 8**

A network of tracks connects two car parks in a festival venue to the exit, as shown in the directed graph below.

2014 Q9.

D 23% correct

B 39% is "not" even a valid cut




The arrows show the direction that cars can travel along each of the tracks and the numbers show each track's capacity in cars per minute.

Five cuts are drawn on the diagram.

The maximum number of cars per minute that will reach the exit is given by the capacity of

A. Cut *A*. ✗

B. Cut *B*. ✗

C. Cut *C*.

D. Cut *D*.

E. Cut *E*.

Cut A = 4+5+5+5+5 = 19.
But is NOT valid, since it doesn't cut car park B from the sink.

Cut B = Not valid.

Cut C = 5 + 12 + 3 = 20

Cut D = 5 + 10 + 3 = 18
Don't count the 4 because it flows back in to the sources.

Cut E = 5 + 11 + 6 = 22

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

# Further Mathematics

## Graded Assessment 3
## WRITTEN EXAMINATION 2
## 2013

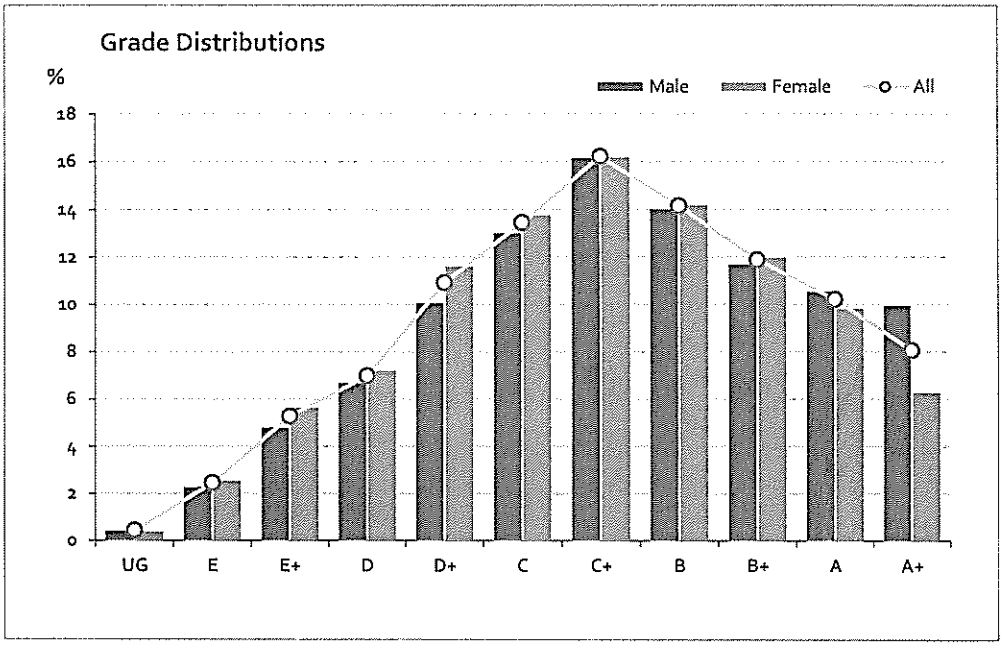
Table of Grade Distributions: male, female and all students assessed

| Grade | | UG | E | E+ | D | D+ | C | C+ | B | B+ | A | A+ | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Male | n | 60 | 307 | 646 | 895 | 1,347 | 1,741 | 2,163 | 1,877 | 1,564 | 1,412 | 1,333 | 13,345 |
| | % | 0 | 2 | 5 | 7 | 10 | 13 | 16 | 14 | 12 | 11 | 10 | 100 |
| Female | n | 61 | 387 | 846 | 1,083 | 1,743 | 2,067 | 2,430 | 2,128 | 1,799 | 1,477 | 946 | 14,967 |
| | % | 0 | 3 | 6 | 7 | 12 | 14 | 16 | 14 | 12 | 10 | 6 | 100 |
| All | n | 121 | 694 | 1,492 | 1,978 | 3,090 | 3,808 | 4,593 | 4,005 | 3,363 | 2,889 | 2,279 | 28,312 |
| | % | 0 | 2 | 5 | 7 | 11 | 13 | 16 | 14 | 12 | 10 | 8 | 100 |
| Cumul | n | 121 | 815 | 2,307 | 4,285 | 7,375 | 11,183 | 15,776 | 19,781 | 23,144 | 26,033 | 28,312 | |
| | % | 0 | 3 | 8 | 15 | 26 | 39 | 56 | 70 | 82 | 92 | 100 | |
| Score Ranges | | 0-5 | 6-15 | 16-24 | 25-32 | 33-42 | 43-53 | 54-66 | 67-78 | 79-89 | 90-101 | 102-120 | Max 120 |

| Summary Statistics: | |
|---|---|
| Mean | 62.3 |
| Std Dev | 26.6 |
| Median | C+ |

| Not Assessed: | | |
|---|---|---|
| Male | n | 1,465 |
| | % | 10 |
| Female | n | 1,057 |
| | % | 7 |
| All | n | 2,522 |
| | % | 8 |

| Total Enrolments incl. Not Assessed: | | |
|---|---|---|
| Male | n | 14,810 |
| Female | n | 16,024 |
| All | n | 30,834 |

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VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

# Further Mathematics

## Graded Assessment 2
## WRITTEN EXAMINATION 1
## 2013

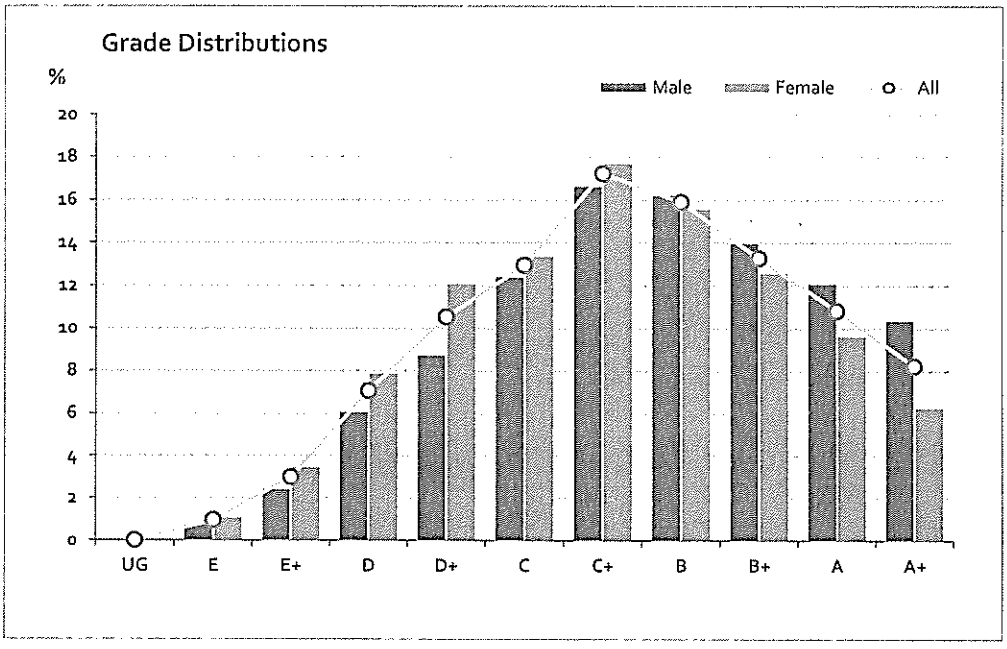
Table of Grade Distributions: male, female and all students assessed

| Grade | | UG | E | E+ | D | D+ | C | C+ | B | B+ | A | A+ | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Male | n | 1 | 104 | 325 | 819 | 1,174 | 1,664 | 2,231 | 2,173 | 1,877 | 1,624 | 1,394 | 13,386 |
| | % | 0 | 1 | 2 | 6 | 9 | 12 | 17 | 16 | 14 | 12 | 10 | 100 |
| Female | n | 0 | 166 | 522 | 1,188 | 1,815 | 2,010 | 2,659 | 2,339 | 1,890 | 1,449 | 947 | 14,985 |
| | % | 0 | 1 | 3 | 8 | 12 | 13 | 18 | 16 | 13 | 10 | 6 | 100 |
| All | n | 1 | 270 | 847 | 2,007 | 2,989 | 3,674 | 4,890 | 4,512 | 3,767 | 3,073 | 2,341 | 28,371 |
| | % | 0 | 1 | 3 | 7 | 11 | 13 | 17 | 16 | 13 | 11 | 8 | 100 |
| Cumul | n | 1 | 271 | 1,118 | 3,125 | 6,114 | 9,788 | 14,678 | 19,190 | 22,957 | 26,030 | 28,371 | |
| | % | 0 | 1 | 4 | 11 | 22 | 35 | 52 | 68 | 81 | 92 | 100 | |
| Score Ranges | | 0-5 | 6-16 | 17-22 | 23-28 | 29-34 | 35-41 | 42-49 | 50-57 | 58-64 | 65-73 | 74-80 | Max 80 |

| Summary Statistics: | |
|---|---|
| Mean | 49 |
| Std Dev | 15.9 |
| Median | C+ |

| Not Assessed: | | |
|---|---|---|
| Male | n | 1,424 |
| | % | 10 |
| Female | n | 1,039 |
| | % | 6 |
| All | n | 2,463 |
| | % | 8 |

| Total Enrolments incl. Not Assessed: | | |
|---|---|---|
| Male | n | 14,810 |
| Female | n | 16,024 |
| All | n | 30,834 |

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