**Further Maths**

**Core: Recursion & Financial Modelling**

**Practice SAC**

**Question 1**

Jeremy is planning to save money for his “Schoolies” activity at the end of the year. He has an account with $1500 in it already, and each month he will add another $75 from his part-time job.

The account pays 4.8% p.a. interest, compounded monthly.

a) What is the interest rate per month? (1 mark)

*Interest rate per month : =  = 0.4 % = 0.004*

b) Using *Vn* to represent the balance of the account after *n* months, write a recurrence relation to model this investment. (2 marks)

*Recurrence relationship: V0 = $1500 Vn+1 = 1.004Vn+ 75*

c) Use your calculator to determine recursive the value of the investment after Jeremy has made six payments. (2 marks)

|  |  |  |
| --- | --- | --- |
| **Month number** | **Calculation details** | **Value of account** |
| 0 |  | 1500.00 |
| 1 | *1500 × 1.004 + 75* | *1581.00* |
| 2 | *1581.00 × 1.004 + 75* | *(1662.324…) 1662.32* |
| 3 | *1662.324… × 1.004 + 75* | *(1743.9732…) 1743.97* |
| 4 | *1743.9732… × 1.004 + 75* | *(1825.9491…) 1825.95* |
| 5 | *1825.9491… × 1.004 + 75* | *(1908.2529…) 1908.25* |
| 6 | *1908.2529… × 1.004 + 75* | *(1990.8859…) 1990.89* |

*The value of the investment after six payments is $1990.89.*

d) Jeremy will close the account after twelve (12) months. How much money, correct to the nearest cent, will he have for his “Schoolies” activities? (1 mark)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **N** | **I(%)** | **PV** | **Pmt** | **FV** | **PpY** | **CpY** |
| *12* | *4.8* | *– 1500* | *– 75* | *0* | *12* | *12* |

*Solve for* ***FV*** *: 2493.671…*

*He will have $2493.67 to spend.*

**Question 2**

Jeremy’s Aunt Alice is about to retire and is wondering what to do with her superannuation money

One option is to invest her $625 000 in an annuity paying 4.5% p.a. compounding monthly.

a) How much would she receive per month if the annuity is for 20 years? (1 mark)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **N** | **I(%)** | **PV** | **Pmt** | **FV** | **PpY** | **CpY** |
| *240* | *4.5* | *– 625 000* | *0* | *0* | *12* | *12* |

*Solve for* ***Pmt*** *: 3954.058…*

*She will receive $3954.06 per month.*

b) If she decided that she wanted to receive $4000 per month, how many fewer payments would she receive from the annuity? (2 marks)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **N** | **I(%)** | **PV** | **Pmt** | **FV** | **PpY** | **CpY** |
| *0* | *4.5* | *– 625 000* | *4000* | *0* | *12* | *12* |

*Solve for N : 235.570…*

*The annuity will last for 236 months, which is four (4) months fewer than in (a)*

c) Another annuity company claims that their annuity product would give her $4250 per month for twenty years for the $625 00 investment. What interest rate, correct to two decimal places, would give this return?

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **N** | **I(%)** | **PV** | **Pmt** | **FV** | **PpY** | **CpY** |
| *240* | *0* | *– 625 000* | *4250* | *0* | *12* | *12* |

*Solve for* ***I%*** *: 5.3599…*

*The interest rate applying in this case is 5.36 % per annum*

Alicia believes that either twenty-year annuity discussed above will not last long enough for her, so she looked at investing in a perpetuity.

d) What is the difference between an annuity and a perpetuity? (2 marks)

*A perpetuity is an annuity that provides regular payments that continue forever. An annuity the payments are for a finite time and will eventually stop.*

e) If the interest offered is 5.2 % p.a., compounding monthly, how much will Alicia receive per month, correct to the nearest dollar? (1 mark)

*Interest (payment) =  = $2708.333… ≈ $2708*

f) If Alicia wanted to receive at least $3000 per month from this perpetuity, how much would she need to invest, correct to the nearest thousand dollars? (2 marks)

*Interest (payment) = *

 *principal =  =  ≈ $692 307.692…*

*She would have to invest $693 000 and will receive $3003 per month!*

*(Investing $692 000 would earn less than $3000 per month – $2998.67!)*

**Question 3**

Jayde is a potter and has purchased a pottery kiln for $6500. The kiln can be depreciated using the reducing balance method at the interest rate 17.5% per year.

a) Using *Vn* to represent the value of the kiln after *n* years, write a recurrence relation that models this depreciation situation. (2 marks)

*Recurrence relation : V0 = $6500, Vn+1 = 0.825Vn*

b) Use your calculator to determine recursively the value of the kiln, each year, for the first five years, and fill in the values in the table below. (2 marks)

|  |  |  |
| --- | --- | --- |
| **Month number** | **Calculation details** | **Value of account** |
| 0 |  | 6500.00 |
| 1 | *6500 × 0.825* | *5362.50* |
| 2 | *5362.50 × 0.825* | *(4424.0625…) 4424.06* |
| 3 | *4424.0625… × 0.825* | *(3649.8515…) 3649.85* |
| 4 | *3649.8515… × 0.825* | *(3011.1275…) 3011.13* |
| 5 | *3011.1275… × 0.825* | *(2484.1802…) 2484.18* |

c) If Jayde will trade in her kiln for a new one when this one has a value less than $2500, explain why this will occur after five years. (1 mark)

*At the end of the fifth year, the value of the kiln will be $2484.18, which is less than the trigger value for replacement.*

Jayde has been advised to look at using the prime cost or flat rate depreciation method using a value of 15% of the purchase price.

d) By what amount, correct to the nearest dollar, will the value of the kiln be depreciated each year? (1 mark)

*15% of $8500 = 0.15 × 6500 = $975.00*

e) Using *Vn* to represent the value of the kiln after *n* years, write a recurrence relation that models this depreciation situation. (2 marks)

*V0 = $6500 Vn+1= Vn – 975*

f) How many years will it take for the value of the kiln to depreciate to $2600.

*Using Vn = V0 + nD, where Vn = $2600, V0 = $6500, D = – 975*

 *2600 = 6500 – 975n*

 *975n = 3900*

 *n = 4.0*

*It will take four (4) years to depreciate to a value of $2600.*

One of Jayde’s pottery colleagues depreciates her kiln using the unit cost method at the rate of $5.70 per kiln load, where the average number of loads per year is 175.

g) Using *Vn* to represent the value of the kiln after *n* loads, write a recurrence relation that models this depreciation situation for Jayde. (2 marks)

*V0 = $6500 Vn+1= Vn – 5.70*

h) How long will it take in years, rounded to the nearest whole number, for the value of Jayde’s kiln to depreciate to $2500 if it is depreciated using this method? (2 marks)

*Find the number of loads required, and then the number of years.*

*Using Vn = V0 + nD, where Vn = $2500, V0 = $6500, D = – 5.70*

 *2500 = 6500 – 5.70n*

 *5.70n = 4000*

 *n = 701.754… ≈ 702*

 *Number of years = 702 ÷ 175 = 4.011… ≈ 4 years*

*It will take four (4) years to depreciate to a value of $2500.*

**Question 4**

Sam has recently bought a home. He borrowed $350 000 from a bank which is charging him 6.78% p.a., compounding monthly, to be paid over twenty years?

a) What is the effective interest rate, correct to three significant figures, that Sam will be paying on his loan at 6.78% p.a. compounding monthly? (1 mark)

*reffective* ***=* ** *= 6.994707… ≈ 6.99 %*

b) Use the Finance Solver function on your calculator to determine Sam’s monthly payment, correct to the nearest cent. Write the values used in the table below. (1 mark)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **N** | **I(%)** | **PV** | **Pmt** | **FV** | **PpY** | **CpY** |
| *240* | *6.78* | *350 000* | *0* | *0* | *12* |  |

*Solve for* ***Pmt*** *: 2667.520…*

*The monthly payment is calculated to be $2667.52.*

c) What is the monthly interest rate that Sam will be charged, written as a decimal correct to three decimals places? (1 mark)

*Monthly Interest rate : =  = 0.565 % = 0.00565 %*

d) Using *Vn* to represent the balance of the loan after *n* months, write down a recurrence relation to model this loan situation. (2 marks)

*Recurrence relation: V0 = $350 000 Vn+1 = 1.00565Vn – 2670*

Part of the amortisation table for Sam’s loan is shown below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Payment number** | **Payment made** | **Interest paid** | **Principal reduction** | **Balance of loan** |
| 0 | 0.00 | 0.00 | 0.00 | 350 000.00 |
| 1 | 2670.00 | *1977.50* | 692.50 | 349 307.50 |
| 2 | 2670.00 | 1973.59 | *696.41* | 348 611.09 |
| 3 | 2670.00 | 1969.65 | 700.35 | *347 910.74* |
| 4 | 2670.00 | 1965.70 | 704.30 | 347 206.44 |
| 5 | 2670.00 | 1961.72 | 708.28 | 346 498.16 |
| 6 | 2670.00 | 1957.71 | 712.29 | 345 785.87 |

e) Calculate to the nearest cent:

i) the interest that will be paid with the first payment? (1 mark)

*Interest = $350 000.00 × 0.00565 = $1977.50*

ii) the amount that is reduced from the principal with the second payment? (1 mark)

*Principal reduction = payment – interest = $2670.00 – $1973.59 = $696.41*

iii) the balance of the loan after the third payment is made? (1 mark)

*Balance of loan = previous balance – principal reduction*

 *= $348 611.09 – $700.35 = $347 910.74*

f) Calculate the total amount of money paid by Sam for his first six payments? (1 mark)

*Total paid = 6 × $2670 = $16 020*

g) How much has Sam actually paid off his loan after the sixth payment? (1 mark)

*Amount paid off = $350 000 – $345 785.87 = $4214.13*

h) Calculate the percentage correct to the nearest whole number, of Sam’s first six payments that have been used to pay the interest charges. (1 marks)

 *Percentage (interest) =* $\frac{Interest paid}{Total amount paid}×100=\frac{16020-4214.13}{16020}×100$ *= 73.69% ≈ 74%*

i) The loan is to be repaid with a number of monthly payments of $2670.00 and a final payment that is to be adjusted so that the loan will be fully repaid after exactly 20 years of monthly payments.

Calculate the amount of the final payments, correct to the nearest cent. (3 marks)

Show details of any Finance Solver calculations in the table provided below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **N** | **I(%)** | **PV** | **Pmt** | **FV** | **PpY** | **CpY** |
| *239* | *6.78* | *350 000* | *– 2670* | *0* | *12* | *12* |

*Solve for FV: 1404.2404…*

*The balance owing after the second last payment is $1404.24.*

 *Interest payable for the last payment =  = $7.93*

 *Final payment = $1404.24 + $7.93 = $1412.17*