**Further Maths**

**Core: Recursion & Financial Modelling**

**Practice SAC**

**Question 1**

Jeremy is planning to save money for his “Schoolies” activity at the end of the year. He has an account with $1500 in it already, and each month he will add another $75 from his part-time job.

The account pays 4.8% p.a. interest, compounded monthly.

a) What is the interest rate per month? (1 mark)

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b) Using *Vn* to represent the balance of the account after *n* months, write a recurrence relation to model this investment. (2 marks)

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c) Use your calculator to determine recursive the value of the investment after Jeremy has made six payments. (2 marks)

|  |  |  |
| --- | --- | --- |
| **Month number** | **Calculation details** | **Value of account** |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |

d) Jeremy will close the account after twelve (12) months. How much money, correct to the nearest cent, will he have for his “Schoolies” activities? (1 mark)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **N** | **I(%)** | **PV** | **Pmt** | **FV** | **PpY** | **CpY** |
|  |  |  |  |  |  |  |

**Question 2**

Jeremy’s Aunt Alice is about to retire and is wondering what to do with her superannuation money

One option is to invest her $625 000 in an annuity paying 4.5% p.a. compounding monthly.

a) How much would she receive per month if the annuity is for 20 years? (1 mark)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **N** | **I(%)** | **PV** | **Pmt** | **FV** | **PpY** | **CpY** |
|  |  |  |  |  |  |  |

b) If she decided that she wanted to receive $4000 per month, how many fewer payments would she receive from the annuity? (2 marks)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **N** | **I(%)** | **PV** | **Pmt** | **FV** | **PpY** | **CpY** |
|  |  |  |  |  |  |  |

c) Another annuity company claims that their annuity product would give her $4250 per month for twenty years for the $625 00 investment. What interest rate, correct to two decimal places, would give this return?

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **N** | **I(%)** | **PV** | **Pmt** | **FV** | **PpY** | **CpY** |
|  |  |  |  |  |  |  |

Alicia believes that either twenty-year annuity discussed above will not last long enough for her, so she looked at investing in a perpetuity.

d) What is the difference between an annuity and a perpetuity? (2 marks)

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e) If the interest offered is 5.2 % p.a., compounding monthly, how much will Alicia receive per month, correct to the nearest dollar? (1 mark)

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f) If Alicia wanted to receive at least $3000 per month from this perpetuity, how much would she need to invest, correct to the nearest thousand dollars? (2 marks)

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\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Question 3**

Jayde is a potter and has purchased a pottery kiln for $6500. The kiln can be depreciated using the reducing balance method at the interest rate 17.5% per year.

a) Using *Vn* to represent the value of the kiln after *n* years, write a recurrence relation that models this depreciation situation. (2 marks)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

b) Use your calculator to determine recursively the value of the kiln, each year, for the first five years, and fill in the values in the table below. (2 marks)

|  |  |  |
| --- | --- | --- |
| **Month number** | **Calculation details** | **Value of account** |
| 0 | - |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |

c) If Jayde will trade in her kiln for a new one when this one has a value less than $2500, explain why this will occur after five years. (1 mark)

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Jayde has been advised to look at using the prime cost or flat rate depreciation method using a value of 15% of the purchase price.

d) By what amount, correct to the nearest dollar, will the value of the kiln be depreciated each year? (1 mark)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

e) Using *Vn* to represent the value of the kiln after *n* years, write a recurrence relation that models this depreciation situation. (2 marks)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

f) How many years will it take for the value of the kiln to depreciate to $2600.

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One of Jayde’s pottery colleagues depreciates her kiln using the unit cost method at the rate of $5.70 per kiln load, where the average number of loads per year is 175.

g) Using *Vn* to represent the value of the kiln after *n* loads, write a recurrence relation that models this depreciation situation for Jayde. (2 marks)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

h) How long will it take in years, rounded to the nearest whole number, for the value of Jayde’s kiln to depreciate to $2500 if it is depreciated using this method? (2 marks)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Question 4**

Sam has recently bought a home. He borrowed $350 000 from a bank which is charging him 6.78% p.a., compounding monthly, to be paid over twenty years?

a) What is the effective interest rate, correct to three significant figures, that Sam will be paying on his loan at 6.78% p.a. compounding monthly? (1 mark)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

b) Use the Finance Solver function on your calculator to determine Sam’s monthly payment, correct to the nearest cent. Write the values used in the table below. (1 mark)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **N** | **I(%)** | **PV** | **Pmt** | **FV** | **PpY** | **CpY** |
|  |  |  |  |  |  |  |

c) What is the monthly interest rate that Sam will be charged, written as a decimal correct to three decimals places? (1 mark)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

d) Using *Vn* to represent the balance of the loan after *n* months, write down a recurrence relation to model this loan situation. (2 marks)

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Part of the amortisation table for Sam’s loan is shown below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Payment number** | **Payment made** | **Interest paid** | **Principal reduction** | **Balance of loan** |
| 0 | 0.00 | 0.00 | 0.00 | 350 000.00 |
| 1 | 2670.00 |  | 692.50 | 349 307.50 |
| 2 | 2670.00 | 1973.59 |  | 348 611.09 |
| 3 | 2670.00 | 1969.65 | 700.35 |  |
| 4 | 2670.00 | 1965.70 | 704.30 | 347 206.44 |
| 5 | 2670.00 | 1961.72 | 708.28 | 346 498.16 |
| 6 | 2670.00 | 1957.71 | 712.29 | 345 785.87 |

e) Calculate to the nearest cent:

i) the interest that will be paid with the first payment? (1 mark)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

ii) the amount that is reduced from the principal with the second payment? (1 mark)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

iii) the balance of the loan after the third payment is made? (1 mark)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

f) Calculate the total amount of money paid by Sam for his first six payments? (1 mark)

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g) How much has Sam actually paid off his loan after the sixth payment? (1 mark)

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h) Calculate the percentage correct to the nearest whole number, of Sam’s first six payments that have been used to pay the interest charges. (1 marks)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

i) The loan is to be repaid with a number of monthly payments of $2670.00 and a final payment that is to be adjusted so that the loan will be fully repaid after exactly 20 years of monthly payments.

Calculate the amount of the final payments, correct to the nearest cent. (3 marks)

Show details of any Finance Solver calculations in the table provided below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **N** | **I(%)** | **PV** | **Pmt** | **FV** | **PpY** | **CpY** |
|  |  |  |  |  |  |  |