**Summary — Topic 5: Recurrence relations**

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| **Generating the terms of a sequence defined by a first-order recurrence relation** | * A first-order recurrence relation defines a relationship between two successive terms of a sequence, for example, between:

*un*, the previous term, and *un* + 1, the next term,or*un* − 1, the previous term, and *un*, the next term.* A first-order recurrence relation has two main parts:

*un* + 1 = *un* + *b* (where *b* is a constant) describes the pattern in the sequence*u*1 = 1 is the first or a starting term in the sequence.* First-order recurrence relations can be expressed as follows:

*un* + 1 = 2*un* + 3 *u*0 = 1.It is read as ‘the next term is twice the previous term plus 3, starting at 1’ or*un* + 1 − *un* = 4 *u*1 = 1It is read as ‘the difference between two consecutive terms is 4, starting at 1’. |
| **Starting term** | * A starting term is needed to fully define a sequence. The same pattern but different starting points gives different sets of numbers.

*un* + 1 = *un* + 2, *u*1 = 3 gives 3, 5, 7, 9, 11, . . .*un* + 1 = *un* + 2, *u*1 = 2 gives 2, 4, 6, 8, 10, . . .* *u*0 is used as the first term for situations that are dependent on time.
* *u*1 is used as the first term for situations that are ordinal, such as placings (first place, *u*1, second place, *u*2,third place, *u*3, . . .) or prizes (first prize, second prize, . . .).
 |
| **The relationship between arithmetic sequences and first-order recurrence relations** | * Pronumeral conventions

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| **Term** | **Arithmetic and geometric sequence convention** | **First-order recurrence relation convention** |
| First term | *a* or *u*1 | *u*0 or *u*1 |
| Common difference | *d* | *d* |
| Common ratio | *r* | *R* |

* The common difference, *d* = *u*2 − *u*1 = *u*3 − *u*2 = *u*4 − *u*3 = . . .
* An arithmetic sequence with a common difference of *d* may be defined by a first-order recurrence relation of the form:

*un* + 1 = *un* + *d* (or *un* + 1 − *un* = *d*)where *d* is the common difference and for*d*> 0 it is an increasing sequence*d* < 0 it is a decreasing sequence. |
| **The relationship between geometric sequences and first-order recurrence relations** | * The geometric common ratio, *r*, is the pronumeral *R* in first order difference equations.
* The common ratio,  . . .
* A geometric sequence with a common ratio of *R* may be defined by a first-order recurrence relation of the form:

*un* + 1 = *Run*where *R* is the common ratio*R* > 1 is an increasing sequence0 < *R* < 1 is a decreasing sequence*R* < 0 is a sequence alternating between positive and negative values. |
| **Interpretation of the graphs of first-order recurrence relations** | * A straight line or linear pattern is given by first-order recurrence relations of the form *un* + 1 = *un* + *d*,  *u1 = a*

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**Summary — Topic 6: Interest and depreciation**

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| **Simple interest** | *

where  ($) is the value of the investment after  time periods($)is the initial amount is the simple interest charged or earned ($)*

where  ($) is the value of the investment after  time periods is the amount of interest earned per period($)is the initial amount  is the interest rate.* Interest rate, , and time period, , must be stated and calculated in the same time terms.
* To find the principal
* To find the interest rate
* To find the period of the loan or investment
 |
| **Compound interest** | * Compound interest is calculated using the formula:

where  ($) is the value of the investment after  time periods($)is the initial amount *R* is the growth or compounding factor  * For compound interest,
* Finance Solver on CAS can be used to solve compound interest problems, especially calculations to find time.
 |
| **Depreciation** | * There are 3 methods by which depreciation can be calculated:

1. flat rate depreciation2. reducing balance depreciation3. unit cost depreciation.* An item is *written off* when its book value becomes *zero*.
* *Scrap value* is an item’s book value when it is no longer used.
 |
| **Flat rate depreciation** |  where  is the value of the asset after *n* depreciating periods*d* is the depreciation each time period. |
| **Reducing balance depreciation** | * Reducing balance depreciation is an example of exponential decay and is calculated using the formula:

 where is the value of the asset after *n* depreciating periods*.** where *r* is the depreciation rate*.* To find time in reducing balance depreciation use Finance Solver on CAS.
 |
| **Unit cost depreciation** | * Unit cost depreciation is based on how much an item is used.

 where is the value of the asset after *n* outputs *d* is the depreciation per output*.* |

**Summary — Topic 7: Loans investments and asset values**

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| **Reducing balance loans — the annuities formula** | * This is the recurrence relation that calculates the value of an annuity after each time period.

 where: = Amount left after *n*+1 payments  = Amount at time *n* *R* =  where *r* is the interest rate per period= Payment amount* The amount owing in a loan account for *n* repayments is given by the annuities formula:

 where: = the amount borrowed (principal) = the compounding interest or growth factor for the amount borrowed =  (*r* = the interest rate per repayment period)  *d* = the amount of the regular payments made per period *n* = the number of payments = the amount owing after *n* payments |
| **Reducing balance loans — further calculations** | **Number of repayments*** Finance Solver on CAS is used to find the number of repayments required to repay a loan in full.

**Effects of changing the repayment*** Increasing the size of the repayment decreases the amount of interest paid and decreases the term of the loan.

**Frequency of repayments*** Increasing the frequency of the repayment decreases the total interest paid and may decrease the term of the loan.

**Changing the rate*** Increasing the interest rate increases the total interest paid and generally increases the term of the loan.
 |
| **Reducing balance and flat rate loan comparisons** | * Reducing balance loans are of greater financial benefit to a borrower than flat rate loans since interest for:

(a) reducing balance loans is calculated on the amount outstanding each period, which continually decreases throughout the life of the loan(b) flat rate loans is calculated on the amount borrowed.* Only the effective interest rate can be directly compared with a compound interest rate as they both take into account the reduction of the principal amount.
* For flat rate loans
* For reducing balance loans

 Interest charged = total repaid − amount borrowed |
| **Effective rate of interest** | * The effective interest rate is a true indication of the interest rate on a loan. It is calculated using a flat interest rate when the loan is progressively being reduced, such as in hire-purchases.
* To calculate the effective annual interest rate, use the formula:

 where*r* = the effective annual interest rate*i* = the nominal rate, as a decimal*n* = the number of compounding periods per year  |
| **Perpetuities** | * A perpetuity is an annuity where a permanently invested sum of money provides regular payments that continue forever.
* The perpetuity formula is:

 where: *d* = the amount of the regular payment per period ($)= the principal ($) *r* = the interest rate per period (%).*Notes*1. The period of the regular payment must be the same as the period of the given interest rate.
2. Finance Solver can be used in calculations involving perpetuities. As the principal does not change, the present value (PV: or negative cash flow) and the future value (FV: or positive cash flow) are entered as the same amount, but with opposite signs.
* The perpetuity formula can be transposed to:

 and  * When using the perpetuity formula, the number of payments each year must be the same as the compounding period of the given interest rate.
* Finance Solver can be used in calculations involving perpetuities. Finance Solver cannot be used if the principal is unknown.
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| **Annuity investments** | * An annuity investment is an investment where an initial sum and regular deposits are made. The interest earned is calculated regularly on the balance of the investment, which increases with each regular deposit (annuity).
* Superannuation is a type of annuity investment.
* The money that accumulates in an annuity investment (or superannuation fund) can be calculated using the formula:

 where: = Amount after n + 1 payments = Amount at time n *r* = Interest rate per period *d* = Deposit amount |