VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
Year

FURTHER MATHEMATICS

Written examination 1

Monday 12 September 2016

Reading Time 1:30 to 1:45 (15 minutes)

Writing Time 1:45 to 3:15 (1 hour 30 minutes)

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

SOLUTIONS PWE 2016.

• Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used. • Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.
Materials supplied • Question book of 32 pages. • Formula sheet. • Answer sheet for multiple-choice questions. • Working space is provided throughout the book.
Instructions • Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this. • Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
At the end of the examination • You may keep this question book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Core

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1

The following stem plot shows the areas, in square kilometres, of 27 suburbs of a large city.

key: 1|6 = 1.6 km^2

1	5 6 7 8
2	1 2 4 5 6 8 9 9
3	0 1 1 2 2 8 9
4	0 4 7
5	0 1
6	1 9
7	
8	4

median position $= \frac{n+1}{2}$

$= \frac{27+1}{2}$

$= 14^{th}$ position.

The median area of these suburbs, in square kilometres, is

A. 3.0
B. 3.1
C. 3.5
D. 30.1
E. 30.5

Question 2

The time spent by shoppers at a hardware store on a Saturday is approximately normally distributed with a mean of 31 minutes and a standard deviation of 6 minutes.
If 2850 shoppers are expected to visit the store on a Saturday, the number of shoppers who are expected to spend between 25 and 37 minutes in the store is closest to

A. 16
B. 68
C. 460
D. 1900
E. 2400

mean $= \mu = 31$

std.dev. $= \sigma = 6$

1 standard deviation is $\mu - \sigma \rightarrow \mu + \sigma$

$31 - 6 \rightarrow 31 + 6$

so $25 \rightarrow 37$ is 68%

68% of 2850 shoppers $= 1938$

Use the following information to answer Questions 3–6.

The following table shows the data collected from a random sample of seven drivers drawn from the population of all drivers who used a supermarket car park on one day. The variables in the table are:

- *distance* – the distance that each driver travelled to the supermarket from their home
- *sex* – the sex of the driver (female, male)
- *number of children* – the number of children in the car
- *type of car* – the type of car (sedan, wagon, other)
- *postcode* – the postcode of the driver's home.

Distance (km)	*Sex* (F = female, M = male)	*Number of children*	*Type of car* (1 = sedan, 2 = wagon, 3 = other)	*Postcode*
4.2	F	2	1	8148
0.8	M	3	2	8147
3.9	F	3	2	8146
5.6	F	1	3	8245
0.9	M	1	3	8148
1.7	F	2	2	8147
2.5	M	2	2	8145

Question 3

The mean, $\bar{x}$, and the standard deviation, s_x, of the variable, distance, for these drivers are closest to

A. $\bar{x} = 2.5$ $s_x = 3.3$
B. $\bar{x} = 2.8$ $s_x = 1.7$
C. $\bar{x} = 2.8$ $s_x = 1.8$
D. $\bar{x} = 2.9$ $s_x = 1.7$
E. $\bar{x} = 3.3$ $s_x = 2.5$

Use CAS.

$\bar{x} = 2.8$, $s_x = 1.82$

Question 4

The number of discrete numerical variables in this data set is

A. 0
B. 1
C. 2
D. 3
E. 4

- Distance → continuous, numerical
- Male/female not numerical
- Number of children is discrete numerical (discrete or whole numbers).
- Type of car is a code
- Post code is a code

only 1

Question 5

The number of ordinal variables in this data set is

A. 0
B. 1
C. 2
D. 3
E. 4

None.

Think! Definition of ordinal is "data that can be arranged into categories that have an order" (ie levels of education).

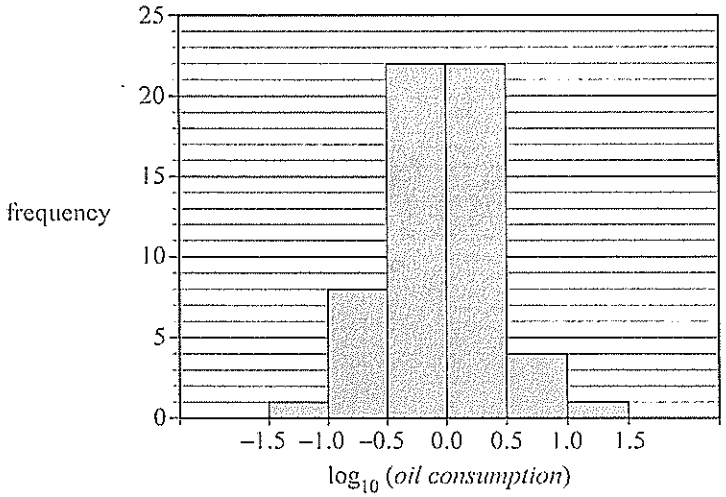
Question 6

The number of female drivers with three children in the car is

A. 0
B. 1
C. 2
D. 3
E. 4

Look for Female "AND" 3 kids.
This occurs once!

Question 7



The histogram above displays the distribution of the annual per capita *oil consumption* (tonnes) for 58 countries plotted on a **log** scale.

The percentage of countries with an annual per capita *oil consumption* of more than 10 tonnes is closest to

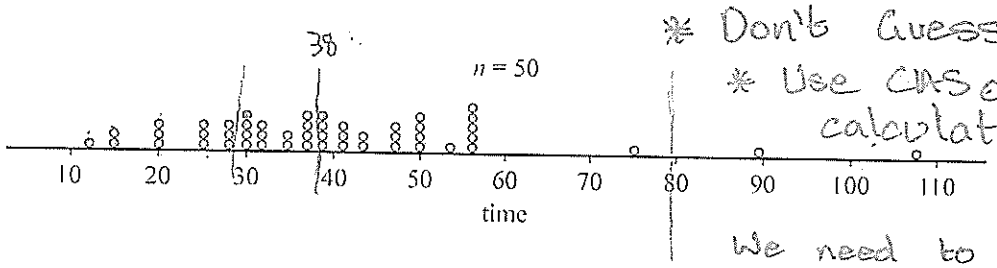
A. 1%
B. 2%
C. 27%
D. 57%
E. 98%

Think! This is tricky!
1. Oil consumption of 10 tonnes will be $\log_{10}(10) = 1$ on this graph.
2. How often is the consumption 1 (what is the frequency) its once out of 58 countries.

$\frac{1}{58} \approx 2\%$

Question 8

The dot plot below shows the distribution of the time, in minutes, that 50 people spent waiting to get help from a call centre.

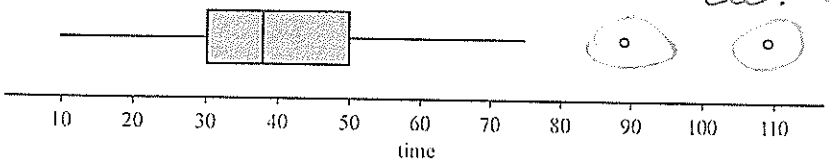


* Don't Guess!!
* Use CAS or Manually calculate.

We need to check for outliers, which means we need Q_1, Q_3 for IQR etc.

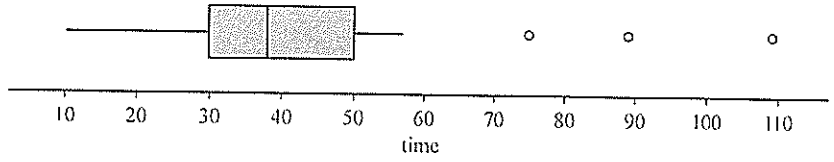
Which one of the following boxplots best represents the data?

A.

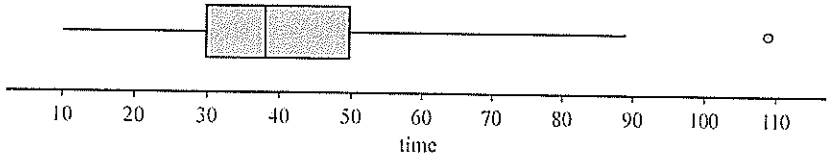


Use CAS!!
- List and spreadsheet and boxplot.
- Either enter all data points or use column of frequencies.

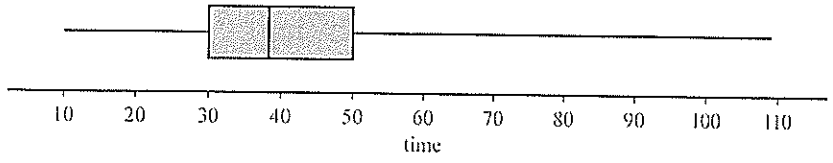
B.



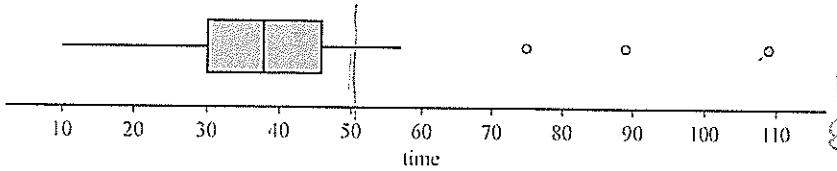
C.



D.



E.



Read from graph!

IQR = 20

$Q_3 = 5$.

Upper fence = $Q_3 + 1\frac{1}{2} \times IQR$

$= 50 + 1\frac{1}{2} \times 20$

Upper fence = 80

IQR = 20

Manually: median = 25.5 position

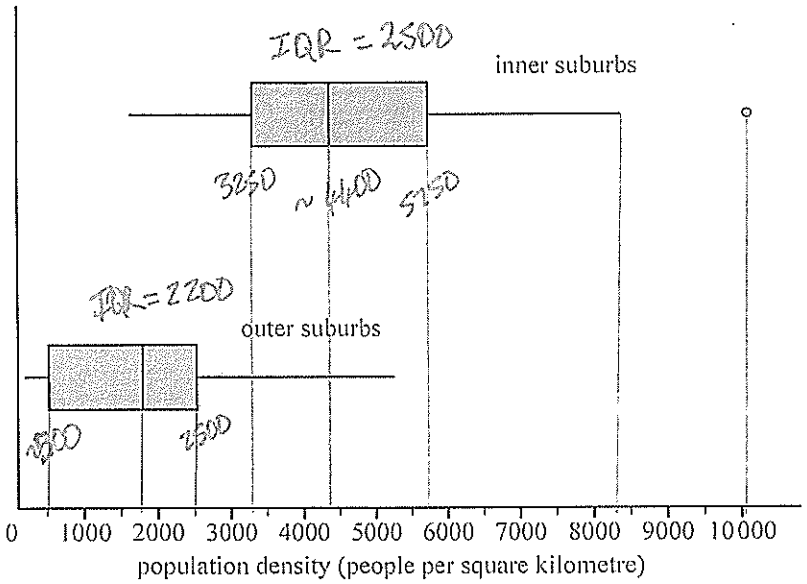
so Q_1 is 13th position = 30

Q_3 is 37th position = 50

Question 9

The parallel boxplots below summarise the distribution of population density, in people per square kilometre, for the inner suburbs and the outer suburbs of a large city.

Check all answers!



Which one of the following statements is **not** true?

A. More than 50% of the outer suburbs have population densities below 2000 people per square kilometre. — True. median is ~1750

B. More than 75% of the inner suburbs have population densities below 6000 people per square kilometre. — $Q_3 = 5750$

C. Population densities are more variable in the outer suburbs than in the inner suburbs. IQR. — True.

D. The median population density of the inner suburbs is approximately 4400 people per square kilometre.

E. Population densities are, on average, higher in the inner suburbs than in the outer suburbs.

↳ True — median inner = ~4400, outer = ~1700. — median ~4400 TRUE.

Question 10

A single back-to-back stem plot would be an appropriate graphical tool to investigate the association between a car's speed, in kilometres per hour, and the

A. driver's age, in years. — numerical.

B. car's colour (white, red, grey, other). — 4 categories

C. car's fuel consumption, in kilometres per litre. — numerical

D. average distance travelled, in kilometres. — numerical

E. driver's sex (female, male). — 2 categories.

back-to-back stem plot is used for numerical and two categories.

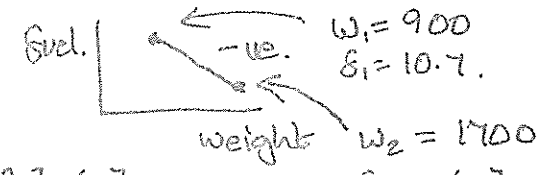
Question 11

The equation of a least squares regression line is used to predict the fuel consumption, in kilometres per litre of fuel, from a car's weight, in kilograms.

This equation predicts that a car weighing 900 kg will travel 10.7 km per litre of fuel, while a car weighing 1700 kg will travel 6.7 km per litre of fuel.

The slope of this least squares regression line is closest to

A. –200.0
B. –0.005
C. –0.004
D. 0.005
E. 200.0



$m = \frac{10.7 - 6.7}{900 - 1700}$

$= -\frac{4}{800} = -0.005$.

Question 12

A large study of secondary-school male students shows that there is a negative association between the time spent playing sport each week and the time spent playing computer games. check all options.

From this information, it can be concluded that

A. male students who spend a lot of time playing computer games do not play sport. ✗
B. encouraging male students to spend less time playing sport will increase the time they spend playing computer games. ✗
C. encouraging male students to spend more time playing sport will reduce the time they spend playing computer games. ✗
D. male students who tend to spend more time playing sport tend to spend less time playing computer games.
E. male students who tend to spend more time playing sport tend to spend more time playing computer games. ✗

Question 13

The seasonal index for heaters in winter is 1.25

To correct for seasonality, the actual heater sales in winter should be

A. reduced by 20%
B. increased by 20%
C. reduced by 25%
D. increased by 25%
E. reduced by 75%

Deseasonalised value = $\frac{\text{actual value}}{\text{seasonal index}}$

$= \frac{\text{actual value}}{1.25}$

$= \frac{1}{1.25} \times \text{actual value}$

$= 0.8 \times \text{actual value}$.

which is the same as reducing it by 20%

Option A

Use the following information to answer Questions 14 and 15.

The seasonal indices for the first 11 months of the year for sales in a sporting equipment store are shown in the table below.

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
Seasonal index	1.23	0.96	1.12	1.08	0.89	0.98	0.86	0.76	0.76	0.95	1.12	

Question 14

The seasonal index for December is

A. 0.89
B. 0.97
C. 1.02
D. 1.23
E. 1.29

Think! Sum of all seasonal indices must be 12 in this case so sum.

Dec = 12 − Σ of others

= 12 − 10.71 = 1.29.

Question 15

In May, the store sold $213 956 worth of sporting equipment.

The deseasonalised value of these sales was closest to

A. $165 857
B. $190 420
C. $209 677
D. $218 322
E. $240 400

deseasonalised = actual value / seasonal index

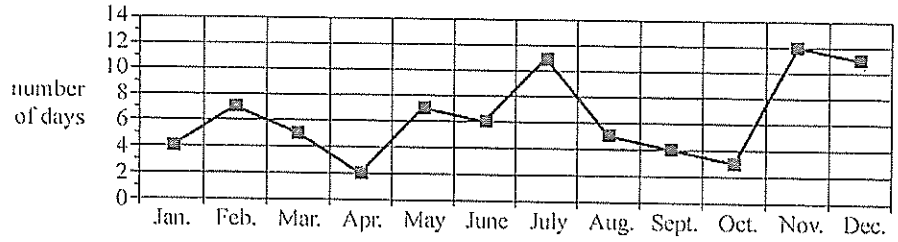
= 213956 / 0.89

= $240,400

option E.

Question 16

The time series plot below shows the number of days that it rained in a town each month during 2011.

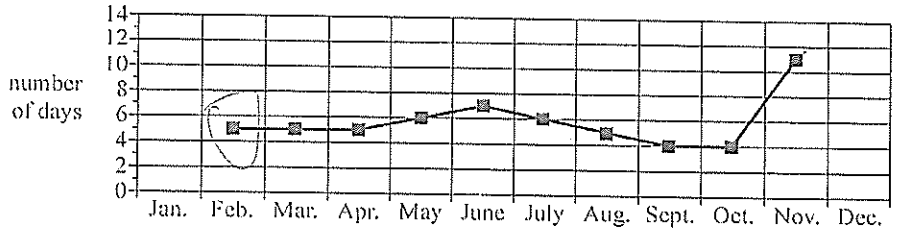


5, 4, 3, 12, 11 -
3, 4, 5, 11, 12.
median 5.

Using five-median smoothing, the smoothed time series plot will look most like

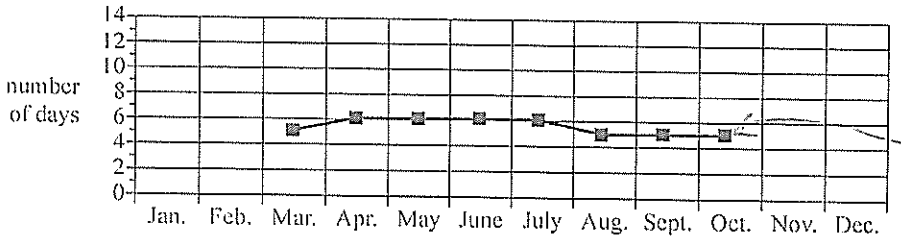
Five median smoothing loses first 2 and last 2 values.

A.



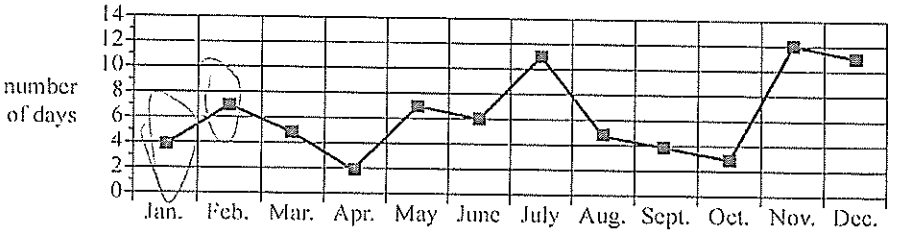
X

B.



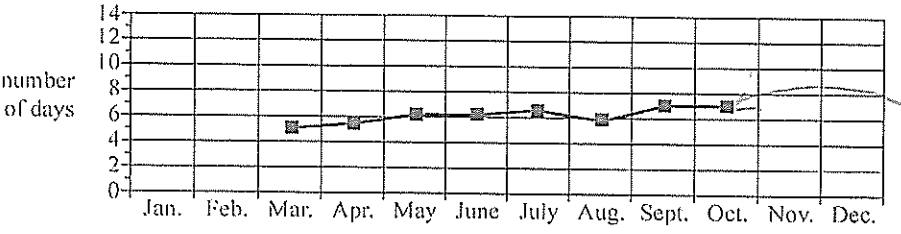
check last 5,
5 median

C.



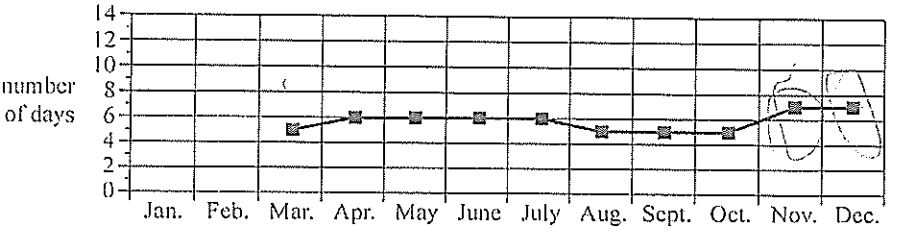
X

D.



check last 5,
7 ← median

E.



X

Recursion and financial modelling

Question 17

$$P_0 = 2000,\ P_{n+1} = 1.5P_n - 500$$

The first three terms of a sequence generated by the recurrence relation above are

A. 500, 2500, 2000 ...
B. 2000, 1500, 1000 ...
C. 2000, 2500, 3000 ...
D. 2000, 2500, 3250 ...
E. 2000, 3000, 4500 ...

Use CAS. $P_0 = 2000$

2000, 2500, 3250, 4375.

Question 18

Which of the following recurrence relations will generate a sequence whose values decay geometrically?

A. $L_0 = 2000,\ L_{n+1} = L_n - 100$ ← down linear
B. $L_0 = 2000,\ L_{n+1} = L_n + 100$ ← goes up
C. $L_0 = 2000,\ L_{n+1} = 0.65L_n$ ← goes down geo.
D. $L_0 = 2000,\ L_{n+1} = 6.5L_n$ ← goes up.
E. $L_0 = 2000,\ L_{n+1} = 0.85L_n - 100$ ← combo down.

Geometric decay means it goes down and it is a multiplication or division.

Question 19

Eva has $1200 that she plans to invest for one year.

One company offers to pay her interest at the rate of 6.75% per annum compounding daily. ← 365 days.

The effective annual interest rate for this investment would be closest to

A. 6.75%
B. 6.92%
C. 6.96%
D. 6.98%
E. 6.99%

OR use CAS

eff(6.75, 365) = 6.98%.

$$r = \left(1 + \frac{i}{n}\right)^n - 1$$

$$r = \left(1 + \frac{0.0675}{365}\right)^{365} - 1$$

$$= 0.0698 = 6.98\%$$

$i = 6.75\%$

$i = \frac{6.75\%}{100}$

$i = 0.0675$

* Note i is decimal NOT percentage.

Question 20

Rohan invests $15 000 at an annual interest rate of 9.6% compounding monthly.

Let V_n be the value of the investment after n months.

A recurrence relation that can be used to model this investment is

A. $V_0 = 15\,000,\ V_{n+1} = 0.96V_n$
B. $V_0 = 15\,000,\ V_{n+1} = 1.008V_n$
C. $V_0 = 15\,000,\ V_{n+1} = 1.08V_n$
D. $V_0 = 15\,000,\ V_{n+1} = 1.0096V_n$
E. $V_0 = 15\,000,\ V_{n+1} = 1.096V_n$

$V_{n+1} = V_0 R$, $R = \left(1 + \frac{r}{100}\right)$

so, $V_{n+1} = V_n \times \left(1 + \frac{r}{100}\right)$

$V_{n+1} = V_n \times \left(1 + \frac{9.6}{12 \times 100}\right)$

$V_{n+1} = V_n \times 1.008$

Use the following information to answer Questions 21–23.

Kim invests $400 000 in an annuity paying 3.2% interest per annum.
The annuity is designed to give her an annual payment of $47 372 for 10 years.
The amortisation table for this annuity is shown below.
Some of the information is missing.

Payment number (n)	Payment made	Interest earned	Reduction in principal	Balance of annuity
0	0	0.00	0.00	400 000.00
1	47 372.00	12 800.00	34 572.00	Q21
2	47 372.00	11 693.70	35 678.30	329 749.70
3	47 372.00	10 551.99	36 820.01	292 929.69
4	47 372.00	9 373.75	37 998.25	254 931.44
5	47 372.00	8 157.81	Q22	215 717.24
6	47 372.00	6 902.95	40 469.05	175 248.19
7	47 372.00	5 607.94	41 764.06	133 484.14
8	47 372.00			90 383.63
9	47 372.00	2 892.28	44 479.72	45 903.90
10	47 372.00	1 468.92	45 903.08	0.83

Question 21

The balance of the annuity after one payment has been made is

A. $339 828.00
B. $352 628.00
C. $365 428.00
D. $387 200.00
E. $400 000.00

400 000 − 34 572.00
= 365 428.00

Question 22

The reduction in the principal of the annuity after payment number 5 is

A. $36 820.01
B. $37 998.25
C. $39 214.19
D. $40 469.05
E. $41 764.06

254 931.44 − 215 717.24
= 39 214.2

Question 23

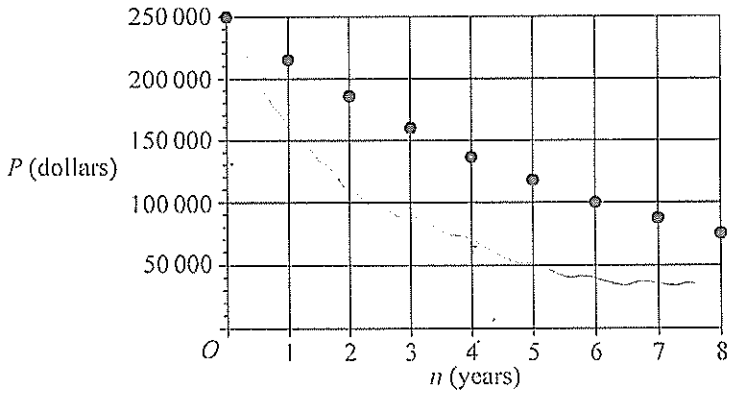
The amount of payment number 8 that is the interest earned is closest to

A. $3799.82
B. $4074.67
C. $4271.49
D. $4836.57
E. $5607.94

3.2% of 133 484.14 = $4 271.49

Question 24

The following graph shows the decreasing value of an asset over eight years.



Let P_n be the value of the asset after n years, in dollars.

A rule for evaluating P_n could be

A. $P_n = 250\,000 \times (1 + 0.14)^n$ ✗

B. $P_n = 250\,000 \times 1.14 \times n$ ✗

C. $P_n = 250\,000 \times (1 - 0.14) \times n$

D. $P_n = 250\,000 \times (0.14)^n$ ~ n=4

E. $P_n = 250\,000 \times (1 - 0.14)^n$ ~

* It is decreasing / decay so, Not A, B.

* The decay is exponential so, Not C.

* The decay is fairly slow, so, not D (which is a fast decay.

END OF SECTION A
TURN OVER

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

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Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents

Module 1 – Matrices

Before answering these questions, you must **shade** the 'Matrices' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

Matrix B, below, shows the number of photography (P), art (A) and cooking (C) books owned by Steven (S), Trevor (T), Ursula (U), Veronica (V) and William (W).

$$B = \begin{array}{c} \\ S \\ T \\ U \\ V \\ W \end{array} \begin{array}{c} \begin{array}{ccc} P & A & C \end{array} \\ \begin{bmatrix} 8 & 5 & 4 \\ 1 & 4 & 5 \\ 3 & 3 & 4 \\ 4 & 2 & 2 \\ 1 & 4 & 1 \end{bmatrix} \end{array}$$

The element in row i and column j of matrix B is b_{ij}.

The element b_{32} is the number of

A. art books owned by Trevor.

B. art books owned by Ursula.

C. art books owned by Veronica.

D. cooking books owned by Ursula.

E. cooking books owned by Trevor.

$b_{3,2}$ is 3rd row (Art books) 2nd column (Ursula).

Question 2

The total cost of one ice-cream and three soft drinks at Catherine's shop is \$9.

The total cost of two ice-creams and five soft drinks is \$16.

Let x be the cost of an ice-cream and y be the cost of a soft drink.

The matrix $\begin{bmatrix} x \\ y \end{bmatrix}$ is equal to

A. $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

B. $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

D. $\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

E. $\begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

Careful!! Its trickie

step 1:

setup simultaneous equation.

$x + 3y = 9$

$2x + 5y = 16$

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 16 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}^{-1}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$

Question 3

Consider the following four statements.

A permutation matrix is always:

I a square matrix ← true.

II a binary matrix ← true.

III a diagonal matrix

IV equal to the transpose of itself.

How many of the statements above are **true**?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 4

Four people, Ash (A), Binh (B), Con (C) and Dan (D), competed in a table tennis tournament.

In this tournament, each competitor played each of the other competitors once.

The results of the tournament are summarised in the matrix below.

A 1 in the matrix shows that the player named in that row defeated the player named in that column. For example, the 1 in row 3 shows that Con defeated Ash.

winner \ loser	A	B	C	D
A	0	1	0	1
B	0	0	1	0
C	1	0	0	0
D	0	1	1	0

1 and 2 step. SUM

$$A^2 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{bmatrix} A & B & C & D \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{array}{l} \\ \rightarrow 5 \\ \rightarrow 2 \\ \rightarrow 3 \\ \rightarrow 4 \end{array}$$

In the tournament, each competitor was given a ranking that was determined by calculating the sum of their one-step and two-step dominances.

The competitor with the highest sum is ranked number one (1). The competitor with the second-highest sum was ranked number two (2), and so on.

Using this method, the rankings of the competitors in this tournament were

A. Dan (1), Ash (2), Con (3), Binh (4).

B. Dan (1), Ash (2), Binh (3), Con (4).

C. Con (1), Dan (2), Ash (3), Binh (4).

D. Ash (1), Dan (2), Binh (3), Con (4).

E. Ash (1), Dan (2), Con (3), Binh (4).

Rank.

Ash(1), Dan(2), Con(3)

Binh(4).

Question 5

The matrix S_{n+1} is determined from the matrix S_n using the rule $S_{n+1} = TS_n - C$, where T, S_0 and C are defined as follows.

$$T = \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.4 \end{bmatrix},\ S_0 = \begin{bmatrix} 100 \\ 250 \end{bmatrix} \text{ and } C = \begin{bmatrix} 20 \\ 20 \end{bmatrix}$$

Use CAS

$S_1 = TS_0 - C = \begin{bmatrix} 180 \\ 130 \end{bmatrix}$

$S_2 = T \times S_1 - C = \begin{bmatrix} 148 \\ 122 \end{bmatrix}$

Given this information, the matrix S_2 equals

A. $\begin{bmatrix} 100 \\ 250 \end{bmatrix}$

B. $\begin{bmatrix} 148 \\ 122 \end{bmatrix}$

C. $\begin{bmatrix} 170 \\ 140 \end{bmatrix}$

D. $\begin{bmatrix} 180 \\ 130 \end{bmatrix}$

E. $\begin{bmatrix} 190 \\ 160 \end{bmatrix}$

Question 6

A and B are square matrices such that $AB = BA = I$, where I is an identity matrix.
Which one of the following statements is not true?

A. $ABA = A$

B. $AB^2A = I$

C. B must equal A

D. B is the inverse of A

E. both A and B have inverses

A B.

$m \times n$ $a \times b$.

$n = a$.

$m \times n$ $n \times b$.

$n \times b$ $m \times n$.

$b = m$.

$n \times m$ $m \times n$.