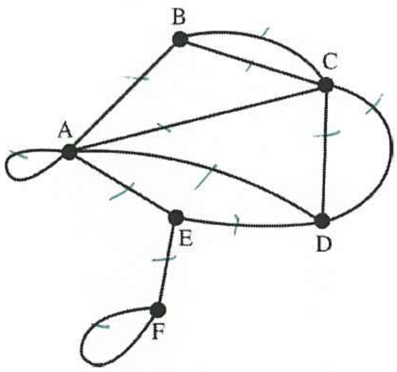
Chapter 5 - Networks Revision

1 From the graph state:
(a) the number of vertices Vertices = 6

(b) the number of edges Edges = 12

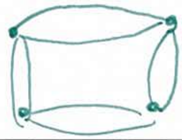
(c) the degree of each vertex A = 6 B = 3 C = 5 d = 4 e = 3 f = 3

(d) whether the graph is connected.



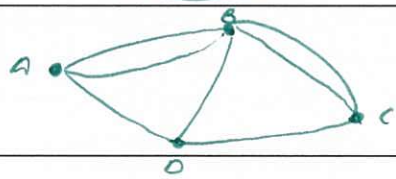
---

2 Draw a graph that has 4 vertices and 8 edges.



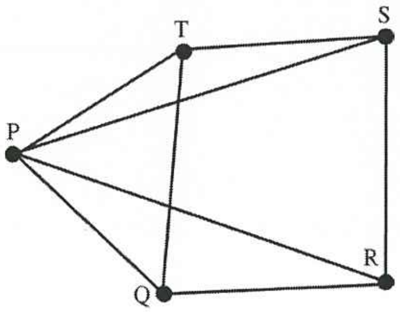
---

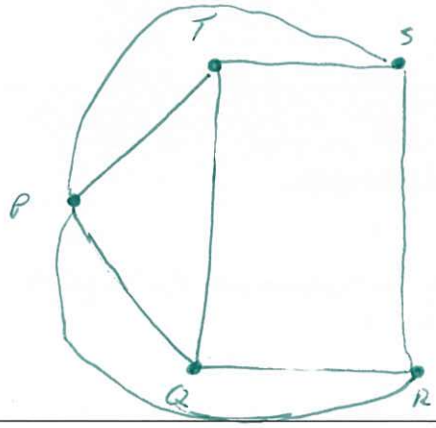
3 Draw the graph that has 4 vertices such that
deg(A) = 3, deg(B) = 5, deg(C) = 2, deg(D) = 2.



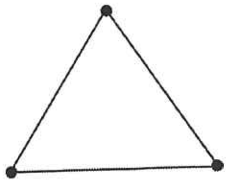
---

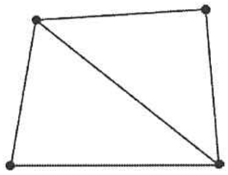
4 Redraw this graph as a planar graph.

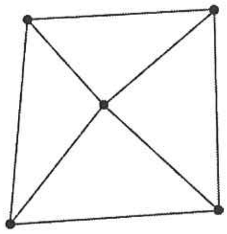




5 Complete the table of vertices (V), regions (R) and edges (E) for the 3 connected planar graphs.







Vertices (V)	Regions (R)	Edges (E)	$V+R-E$
3	2	3	2
4	3	5	2
5	5	8	2

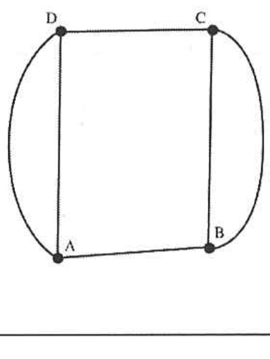
Write down Euler's rule for connected planar graphs.

$V + R - 2 = 2$

6 Use Euler's rule to determine the number of edges in a connected planar graph with 40 vertices and 42 regions.

$40 + 42 - E = 2 \quad 2 = 82 - E, \quad E = 80$

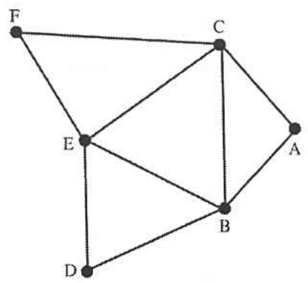
7 Explain why the Eulerian path for this graph could not be drawn.



Because all vertices are of odd degree

· Need 0 or 2 odds degrees to make it work

8 Explain why the Eulerian circuit for this graph is possible.

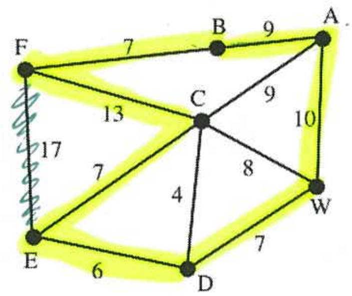


Because all vertices are of even degree.

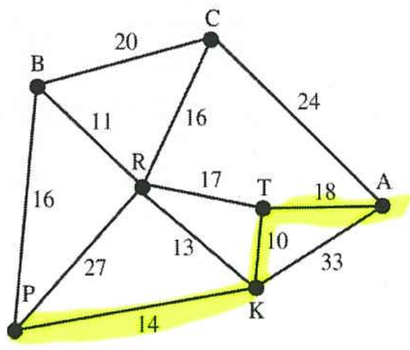
9 A courier supplies, daily, the parcels to outlets A, B, C, D, E and F from a parcel distribution centre W. The distances (in km) along the roads are shown on the graph. Determine the shortest delivery route and the distance to each parcel outlet if the courier returns to the centre.

W - D - E - C - F - B - A - W

= 59



10 Find the shortest distance along the roads between the towns P and A as shown.



P - K - T - A

= 42

Chapter 4 - Matrices Revision

1 Write the order of these matrices:

(a) $\begin{bmatrix} 2 & {}^-3 & 5 \\ 6 & 7 & 1 \end{bmatrix}$ 2×3

(b) $\begin{bmatrix} 8 \\ {}^-4 \\ 2 \end{bmatrix}$ 3×1

2 Matrix $A = \begin{bmatrix} {}^-4 & 10 \\ {}^-6 & 2 \end{bmatrix}$

Find:

(a) $2A$ $\begin{bmatrix} -8 & 20 \\ -12 & 4 \end{bmatrix}$

(b) ${}^-3A$ $\begin{bmatrix} -12 & 30 \\ -18 & 6 \end{bmatrix}$

(c) $\frac{1}{2}A$ $\begin{bmatrix} -2 & 5 \\ -3 & 1 \end{bmatrix}$

(d) $0.3A$ $\begin{bmatrix} -1.2 & 3 \\ -1.8 & 0.6 \end{bmatrix}$

---

3 Given $A = \begin{bmatrix} 1 & {}^-2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} {}^-5 & {}^-1 \\ 6 & 3 \end{bmatrix}$,

find:

(a) $A + B$ $\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -5 & -1 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} -4 & -3 \\ 9 & 7 \end{bmatrix}$

(b) $2A + B$ $2\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -5 & -1 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} -3 & -5 \\ 12 & 11 \end{bmatrix}$

(c) $A + 2B$ $\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} + 2\begin{bmatrix} -5 & -1 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} -9 & -4 \\ 15 & 10 \end{bmatrix}$

---

4 Given $P = \begin{bmatrix} {}^-2 & 3 \\ 5 & 4 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & {}^-4 \\ {}^-3 & 2 \end{bmatrix}$,

find:

(a) $P - Q$ $[P] - [Q] = \begin{bmatrix} -3 & 7 \\ 8 & 2 \end{bmatrix}$

(b) $2P - Q$ $2[P] - [Q] = \begin{bmatrix} -5 & 10 \\ 13 & 6 \end{bmatrix}$

(c) $P - 2Q$ $[P] - 2[Q] = \begin{bmatrix} -4 & 11 \\ 11 & 0 \end{bmatrix}$

---

5 If $\begin{bmatrix} {}^-9 & 4 \\ 8 & {}^-6 \end{bmatrix} + \begin{bmatrix} a & b \\ {}^-10 & 2 \end{bmatrix} = \begin{bmatrix} {}^-6 & {}^-4 \\ {}^-2 & c \end{bmatrix}$, find the values of a, b and c.

$a = 3$ $b = -8$ $c = -4$

---

6 Calculate the following products:

(a) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} ^-2 \\ 4 \end{bmatrix}$ $= \begin{bmatrix} -6 \\ 12 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 0 & ^-1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $= \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

7

(a) Calculate the product $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} ^-4 & 2 \\ 3 & 5 \end{bmatrix}$ $= \begin{bmatrix} -4 & 2 \\ 3 & 5 \end{bmatrix}$

(b) Calculate the product $\begin{bmatrix} 3 & 6 \\ ^-2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ -2 & 4 \end{bmatrix}$

(c) Write the special name of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Identity Matrix

---

8 Given $A = \begin{bmatrix} ^-3 & 1 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 4 \\ ^-7 & 3 \end{bmatrix}$:

(a) calculate the product $A \times B$ $= \begin{bmatrix} -25 & -9 \\ -16 & 20 \end{bmatrix}$

(b) calculate the product $B \times A$ $= \begin{bmatrix} -46 & 22 \\ 0 & 5 \end{bmatrix}$

(c) Is $A \times B = B \times A$? Write your conclusion. No because you're not multiplying the same elements.

---

9 Given $P = \begin{bmatrix} ^-5 & 3 \\ 4 & ^-2 \end{bmatrix}$, $Q = \begin{bmatrix} 2 & 6 \\ ^-1 & 3 \end{bmatrix}$ and

$R = \begin{bmatrix} ^-7 & 3 \\ ^-2 & 4 \end{bmatrix}$:

(a) calculate the product PQ $\begin{bmatrix} -13 & -21 \\ 10 & 18 \end{bmatrix}$

(b) calculate the product QR $\begin{bmatrix} -26 & 30 \\ 1 & 9 \end{bmatrix}$

(c) calculate the product $(PQ)R$ $\begin{bmatrix} 133 & -123 \\ -106 & 102 \end{bmatrix}$

(d) calculate the product $P(QR)$ $\begin{bmatrix} 133 & -123 \\ -106 & 102 \end{bmatrix}$

(e) is $(PQ)R = P(QR)$? Write your conclusion. Yes, it's the same a $P \times Q \times R$

---

10 If $\begin{bmatrix} a & 3 \\ b & ^-2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} ^-3 & 1 \\ 0 & ^-5 \end{bmatrix}$, find the values of a and b.

IGNORE

11 Calculate the product $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 3 & ^-1 \\ ^-5 & 2 \end{bmatrix}$ and write the inverse of $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$.

Product $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

12 (a) Find the determinants of these matrices:

(i) $A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ $(5 \times 2) - (3 \times 3) = 1$

(ii) $B = \begin{bmatrix} ^-5 & 7 \\ ^-2 & 3 \end{bmatrix}$ $(-5 \times 3) - (7 \times -2) = -1$

(iii) $C = \begin{bmatrix} 5 & 3 \\ ^-6 & ^-4 \end{bmatrix}$ $(5 \times -4) - (3 \times -6) - 2$

(iv) $D = \begin{bmatrix} 4 & 2 \\ 10 & 5 \end{bmatrix}$ $(4 \times 5) - (2 \times 10) = 0$

(b) Which special name is given to the matrix D? Singuler Matrix

13 For the matrix $P = \begin{bmatrix} ^-3 & 1 \\ 10 & ^-4 \end{bmatrix}$ find:

(a) det P $(-3 \times -4) - (10 - 1) = 1$

(b) P^{-1}, the inverse of P. $\begin{bmatrix} -2 & -0.5 \\ -5 & -1.5 \end{bmatrix}$

Chapter 6 - Sequences Revision

1 Determine the first 3 terms of the sequence
$t_n = 2n - 6, n \in \{1, 2, 3, \ldots\}$.

$-4, -2, 0$

---

2 Show that the sequence
t_n: $\{^-0.9, ^-0.7, ^-0.5, \ldots\}$ is an arithmetic sequence.

$d = -0.2$

$-0.7 - -0.9 = -0.2 = -0.5 - -0.7 = -0.2$

---

3 Find the rule for the arithmetic sequence
t_n: $\{\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \ldots\}$.

$d = t_3 - t_2 = t_2 - t_1 = \frac{5}{8} - \frac{3}{8} = \frac{3}{8} - \frac{1}{8} = \frac{2}{8}$ or $.25$

---

4 Find the 10th term of the arithmetic sequence
t_n: $\{^-10, ^-6, ^-2, \ldots\}$.

$-10 + (10-1) \times 4 = 26$

---

5 The 3 consecutive terms of an arithmetic sequence are 3.6, y, 8.2. Find the value of y.

$8.2 - 3.6 = \frac{4.6}{2} = 3.2$

$8.2 - 3.6 = 5.9$

$5.9 - 3.6 = 3.6$

so $y = 5.9$

---

6 Insert 3 evenly spaced numbers between $^-2$ and 10.

$10 - -2 = \frac{12}{4} = 3$

$-2, 1, 4, 7, 10$

$d = 3$

---

7 Find the 10th term of the arithmetic sequence where the 1st term is 5 and the 4th term is 17.

$17 - 5 = \frac{12}{4} = 3 \quad d = 3 \quad t_n = 5 + (10-1)3 = 32$

---

8 Find the difference between the 4th term and the 10th term of the arithmetic sequence
t_n: $\{2, ^-1, ^-4, \ldots\}$.

$t_{10} = 2 + (10-1)3$
$= 29$

$t_4 = 2(4-1)3$
$= 11$

difference is
$29 - 11 = 18$

---

9 The 10th term in an arithmetic sequence is 8 and the 4th term is $^-4$. Determine the 1st term a.

$a = t_n - (n-1)d$

$\frac{8 - -4}{6} = 2 \quad d = 2$

$a = 8 - (10-1)2$

$a = -10$

Chapter 1 & 10 - Linear Relations Equations & Linear Graphs Revision

1 Solve for x:
$3x - 4 = 6 - 2x$

$3x = 10 - 2x$ $5x = 10$ $\frac{5x}{5} = \frac{10}{5}$ $x = 2$

---

2 Solve for x:
$3(x - 5) = 3(2x + 6)$

$3x - 15 = 6x + 18$ $3x = 6x + 33$ $-3x = -33$ $x = -11$

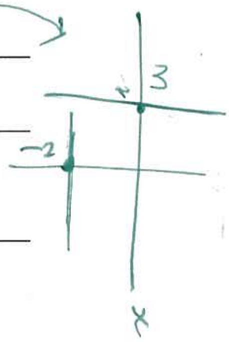
---

3 Solve for x:
$5(3 - 2x) = 20$

$15 - 10x = 20$ $-10x = 5$ $x = -\frac{1}{2}$

---

4 On the same set of axes, sketch the graphs of the lines $x = {}^-2$ and $y = 3$.



---

5 For the equation $5x - 2y + 3 = 0$, find the value of y when $x = {}^-2$.

$-10 - 2y + 3 = 0$ $-2y = 7$ $= -3.5$

---

6 Make y the subject of the equation
$14x - 7y = {}^-21$.

$y = \frac{-21 - 14x}{-7}$

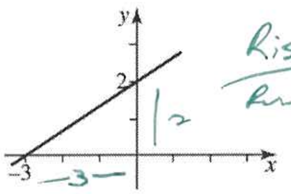
---

7 Complete the following table for the function
$y = 3 - 2x$.

x	$^-3$	$^-2$	2	4
y	-3	-1	-1	-5

---

8 Write the gradient of the line shown.



$\frac{\text{Rise}}{\text{Run}} = \frac{2}{3}$

---

9 A line has a gradient of $-\frac{2}{3}$ and passes through the points (3, 5) and (y, 8). Find the value of y.

$y = 1$

---

10 Find the gradient of the line joining the points ($^-4$, 1) and ($^-2$, 5).

$m = \frac{4}{2} = 2$

---

11 A line with a slope of $\frac{1}{2}$ passes through the point (2, $^-4$). Find its equation.

$y = \frac{1}{2}x - 5$

---

12 Sketch the graph of $y = 2x$ and $y = \dfrac{x}{2}$ on the same set of axes. Show the line $y = x$ also.