Dijkstra's algorithm - Revision

Question 1 The table contains the first line first of a Dijkstra's algorithm solution to a shortest path problem.

	B	*C*	*D*	*E*	*F*
A	5	2	×	×	6

a) Which vertex is the starting vertex for the problem?

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b) Which two vertices are not directly connected to the starting vertex?

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c) Which vertex will be the next vertex in the algorithm?

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d) What is the length of edge *A*–*F*?

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Question 2 A completed table of calculations for the shortest path through a network using Dijkstra's algorithm is shown on the right.

	B	*C*	*D*	*E*	*F*	*G*	*H*	*I*
A	2	[1]	×	5	×	×	×	×
C	[2]	[1]	×	5	4	×	×	×
B	[2]	[1]	6	5	[4]	×	×	×
F	[2]	[1]	6	[5]	[4]	×	8	×
E	[2]	[1]	[6]	[5]	[4]	7	7	×
D	[2]	[1]	[6]	[5]	[4]	[7]	7	×
G	[2]	[1]	[6]	[5]	[4]	[7]	[7]	8
H	[2]	[1]	[6]	[5]	[4]	[7]	[7]	[8]

a) What is the length of the shortest path from *A* to *G*?

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b) What is the shortest path from *A* to *G*?

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c) What is the length of the shortest path from *A* to *I*?

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d) What is the shortest path from *A* to *I*?

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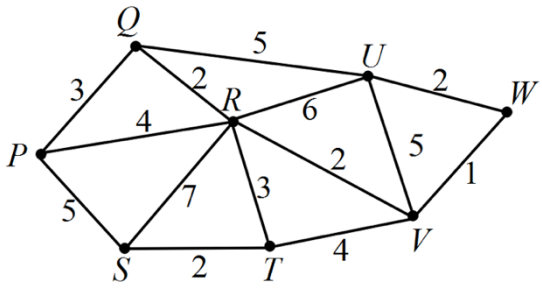
Question 3 The graph below shows the cities *P*, *Q*, *R*, *S*, *T*, *U*, *V* and *W* represented by the vertices and the rail connections between them represented by edges. The numbers on the edges are the times, in hours, it takes to travel by train between each of the cities.



a) Dijkstra's algorithm will be used to find the shortest time to travel by train between *P* and *W*. The first line of the table used to record the calculations for the algorithm is shown below.

	Q	*R*	*S*	*T*	*U*	*V*	*W*
P	[3]	4	5	×	×	×	×

i. Explain why the number 3 in column *Q* has a box around it.

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ii. What does the × mean in the table?

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Two more lines of the algorithm are shown in the table below.

	Q	*R*	*S*	*T*	*U*	*V*	*W*
P	[3]	4	5	×	×	×	×
Q	[3]	[4]	5	×	*a*	×	×
R	[3]	[4]	5	7	8	6	×

iii. Show that the value of *a* is 8.

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iv. What will be the vertex heading of the next row of the table?

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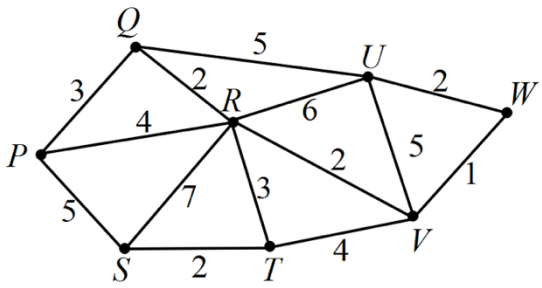
v. Complete Dijkstra's algorithm to find the shortest path from P to W and the shortest time for travelling between P and W by train.

b) Some of the rail connections are considered unnecessary and will be removed so that each town remains connected to all the other towns with the least overall travel time between towns.

i. Draw the minimum spanning tree for the graph of rail connections.



ii. Write down the total weight of the minimum spanning tree.

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iii. Compared to the shortest time to travel between P and W identified by Dijkstra's algorithm, how much longer will be spent travelling between P and W once the rail connections are removed?

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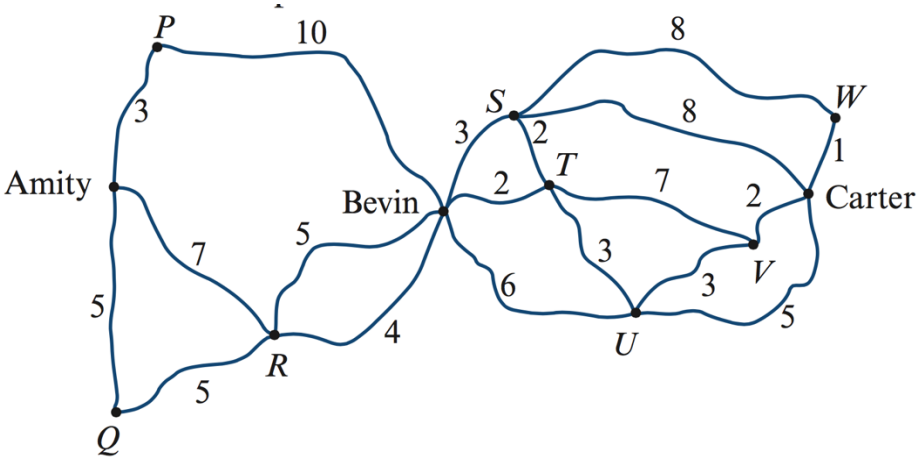
Question 4 The diagram below shows the roads that connect the towns of Amity, Bevin and Carter represented as edges of a network. The vertices of the network, labelled *P*,*Q*,*R*,*S*,*T*,*U*,*V* and *W*, are checkpoints for the Amity Cycling Club road race. The numbers on the edges of the network are the lengths, in kilometres, of the roads between the checkpoints and the towns.



a. Find, by inspection, the length of the shortest path from Amity to Bevin.

A road race for junior riders begins at Amity and ends at Carter. Participants are allowed to take any route they prefer.

b. Use Dijkstra's algorithm to find the shortest path from Bevin to Carter.

c. Using your answers to parts a and e, what is the shortest distance from Amity to Carter?