Point Cook Senior

Further Mathematics 2016
Core: RECURSION AND FINANCIAL MODELLING
Chapter 6 – Interest and depreciation

Key knowledge

- the use of first-order linear recurrence relations to model flat rate and unit cost and reduce balance depreciation of an asset over time, including the rule for the future value of the asset after *n* depreciation periods
- the concepts of financial mathematics including simple and compound interest, nominal

Key skills

- demonstrate the use of a recurrence relation to determine the depreciating value of an asset or the future value of an investment or a loan after *n* time periods, including from first principles for $n \le 5$
- use a rule for the future value of a compound interest investment or loan, or a depreciating asset, to solve practical problems

Chapter Sections	**Questions to be completed**
6.2 Simple interest	1, 2, 3, 4, 6, 7, 8, 9, 11a, 12ab, 13c, 14a, 16, 22
6.3 Compound interest tables	1, 2, 3, 4, 5, 6, 9, 11, 13
6.4 Compound interest formula	1, 3, 5, 6, 7ac, 8ac, 9ab, 10ac, 11, 12, 16ac
6.5 Finding rate or time for compound interest	1, 4, 5, 6, 7, 10, 11ab, 12ab, 13, 15
6.6 Flat rate depreciation	1, 4, 5, 7, 8, 11, 13, 15
6.7 Reducing balance depreciation	2, 3, 6, 7, 8, 10, 12, 14, 16, 18
6.8 Unit cost depreciation	1, 3, 5, 7, 9, 11, 13, 18

More resources available at

http://pcsfurthermaths.weebly.com

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6.2 Simple Interest

When you deposit money into a bank account, the bank is effectively borrowing money from you, which they then use. They pay you a small amount to "thank-you" for letting them use it. This is called interest. If the bank gives you interest of a *fixed amount* at *regular time periods*, this is called a **Simple Interest Investment**.

If you borrow money from the bank and you are charged a *fixed amount* of interest *at regular time periods* it is called a **Simple Interest Loan.**

Simple Interest is an example of linear growth. As we have seen in Chapter 5 linear growth can be expressed as first-order recurrence relation. The amount borrowed or invested is the starting value (V_0) or *Principal*. And with Simple Interest the rule is that a small amount is added at each step. This amount is usually a percentage of the Principal. This percentage is called the *interest rate*, and is expressed over a period of time. For example, 6% per annum (per year) or 0.05% per month etc.

Recurrence relation for Simple Interest

Let V_0 = Principal (the original amount invested)

Let r = be the percentage interest rate

Let V_n be the value of the loan or investment after n years

Simple interest can then be represented by a first-order linear recurrence relation.

$$V_{n+1} = V_n + d, d = \frac{V_0 \times r}{100},$$

where V_n represents the value of the investment after n time periods, d is the amount of interest earned per period, V_0 is the initial (or starting) amount and r is the interest rate.

So, the *Total* amount of a loan or investment is given by:

Total amount of loan or investment = initial amount or principal + interest

$$V_n = V_0 + I$$

where I is the Total Interest earned over the **entire time period**

$$I = \frac{V_0 r n}{100}$$

I = simple interest charged or earned ($)

V_0 = principal (money invested or loaned) ($)

r = rate of interest per period (% per period)

n = the number of periods (years, months, days) over which the agreement operates

Worked Example 1

$325 is invested in a simple interest account for 5 years at 3% p.a.

a) Set up a recurrence relation to find the value of the investment after *n* years.

b) Use the recurrence relation from part (a) to find the value of the investment at the end of each of the first 5 years.

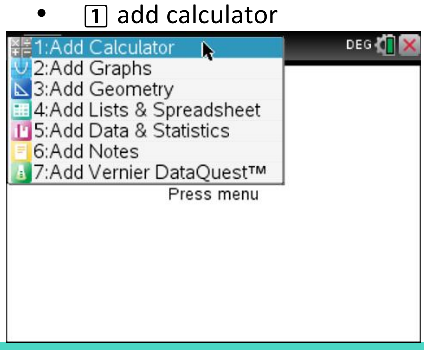
$n + 1$	**V_n($)**	**V_{n+1} ($)**
1	325	
2		
3		
4		
5		

Worked Example 1(b) on CAS calculator

Start with a blank calculator page.

Press

- [on] Home
- [1] New document
- [1] add calculator



Enter the starting value

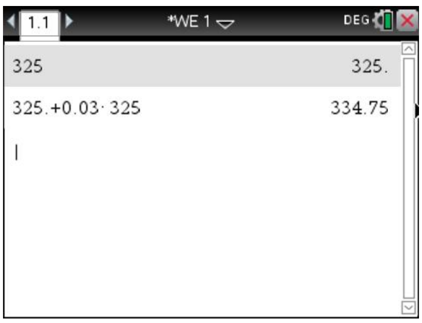
- Type 325 [3][2][5]
- press enter [enter]

Next

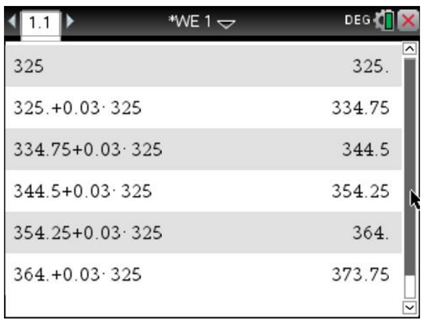
type [+][.][0][3][×][3][2][5]

Press enter [enter]

Note: when you press enter, the CAS converts ANS to the value of the previous answer (in this case 325)



Pressing [enter] repeatedly applies the rule "+0.03x325" to the last calculated value, in the process generating the amount of the investment at the end of each year as shown.

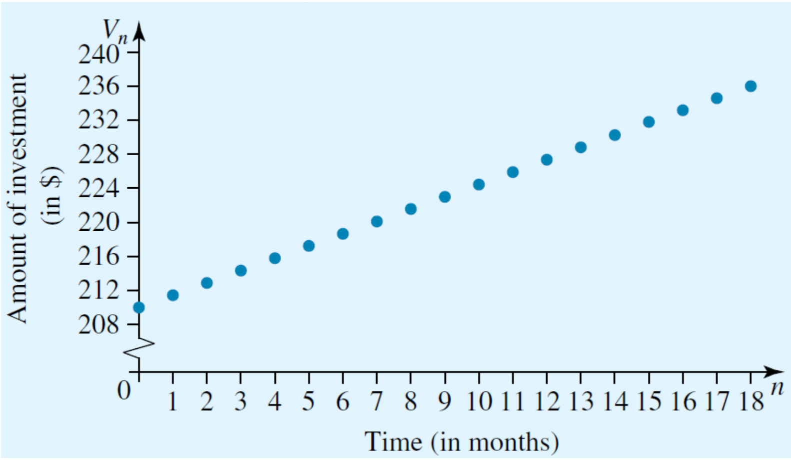


Worked Example 2

Jan invests $210 with building society in a fixed deposit account that paid 8% p.a. simple interest for 18 months.

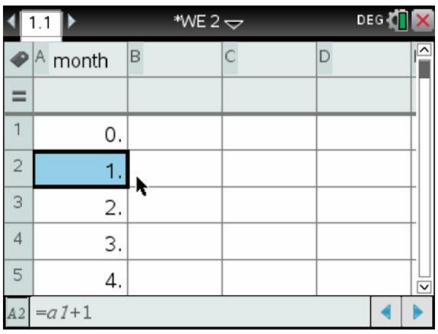
a) How much did she receive after the 18 months?

b) Represent the account balance for each of the 18 months graphically.



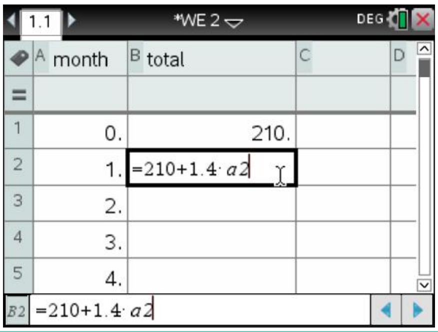
Worked Example 2(b) on CAS Calculator

Label column A “month”
Enter 0 in cell A1
In cell A2 enter: =a1+1
Fill down until the 18th month

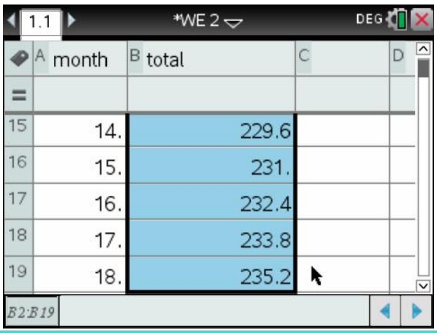


Label column B “total”
Enter $210 in cell b1
In the next cell (B2) enter the equation

=210+1.4 x a2

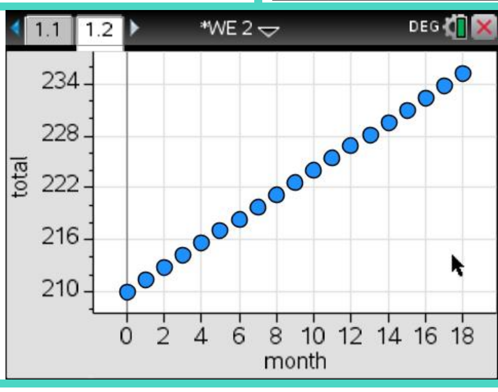


Now fill down this equation to the cells below.
Press
Menu [menu], data [3], fill [3]



Add a data and statistics page
[ctrl] [doc▾]

Put the “month” on the x axis
and “total” on the y axis



Finding V_0, r and n

Transposed simple interest formula

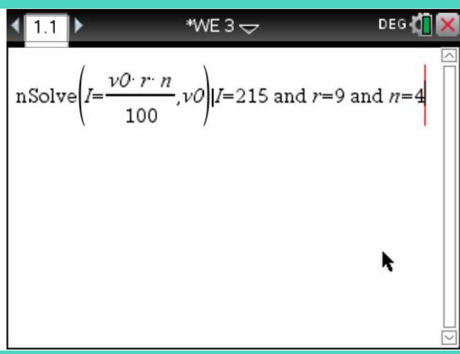
To find the principal:	$V_0 = \frac{100 \times I}{r \times n}$
To find the interest rate:	$r = \frac{100 \times I}{V_0 \times n}$
To find the period of the loan or investment:	$n = \frac{100 \times I}{V_0 \times r}$

Worked Example 3

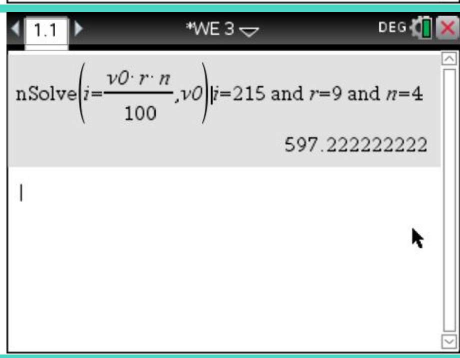
A bank offers 9% p.a. simple interest on an investment. At the end of 4 years the total interest earned was $215. How much was invested?

Worked Example 3 on CAS calculator

On a calculator use the nSolve function, Enter the equation $I = \frac{V_o \times r \times n}{100}$, and set the values of I, r and n using "|"



Press [enter] to get the answer of $597.22

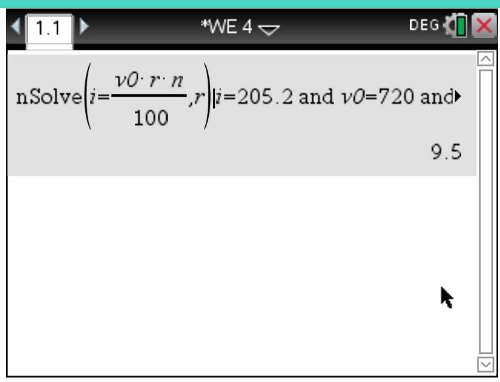


Worked Example 4

When $720 is invested for 36 months it earns $205.20 simple interest. Find the yearly interest rate.

Worked Example 4 on CAS calculator

On a calculator use the nSolve function, Enter the equation $I = \frac{V_o \times r \times n}{100}$, and set the values of I, V_0 and n using "|", I=$205.20, V_0=720 and n=3

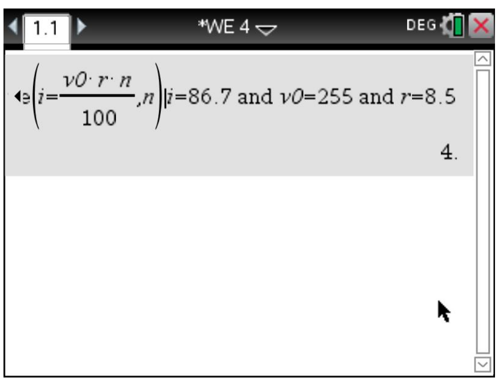


Worked Example 5

An amount of $255 was invested at 8.5% p.a. How long will it take, to the nearest year, to earn $86.70 in interest?

Worked Example 5 on CAS calculator

On a calculator use the nSolve function, Enter the equation $I = \frac{V_o \times r \times n}{100}$, and set the values of I, V_0 and r using "|",I=$86.70, V_0=255 and r=8.5



6.3 Compound Interest Tables

For investments, when interest is added to the initial amount (principal) invested at the end of an interest-bearing period, and then both the principal and interest earn further interest during the next period, which in turn is added to the balance. This process continues for the life of the investment. The interest is said to be **compounded**.

Both the balance of the account and interest increase at regular intervals.

Example

Consider $1000 invested for 4 years at an interest rate of 12% p.a. with interested compounded annually. What will be the final balance of the account?

Time period (n + 1)	V_n($)	Interest ($)	V_{n+1} ($)
1	V_0 = 1000.00	12% of 1000 = 120.00	1000+120 = 1120.00
2	V_1 = 1120.00	12% of 1120 = 134.40	1120+134.40=1254.40
3	V_2 = 1254.40	12% of 1254.40 = 150.53	1254.40+150.53 = 1404.93
4	V_3 = 1404.93	12% of 168.59 = 168.59	1404.93+168.59 = 1573.52
5	V_4 = 1573.52	12% of 1573.52 = 188.82	1573.52+188.82 = 1762.34

So the balance after 5 years is $1762.34.

In the above example the principle is increased by 12% per year. That is at the end of year balance is 112% or 1.12 of the start of year balance.

Time period	Balance($)	
1	1120 = 1000 × 1.12 = 1000 · 1.12	= $1000\,(1.12)^1$
2	1254.40 = 1120 × 1.12 = 1000 × 1.12 × 1.12	= $1000\,(1.12)^2$
3	1404.93 = 1254.40 × 1.12 = 1000 × 1.12 × 1.12 × 1.12	= $1000\,(1.12)^3$
4	1573.52 = 1404.93 × 1.12 = 1000 × 1.12 × 1.12 × 1.12 × 1.12	= $1000\,(1.12)^4$
5	1762.34 = 1573.52 × 1.12 = 1000 × 1.12 × 1.12 × 1.12 × 1.12 × 1.12	= $1000\,(1.12)^5$

If this investment continued for n years, the final balance should be:

$$V_n = 1000\,(1.12)^n = 1000\,(1 + 0.12)^n = 1000\left(1 + \frac{12}{100}\right)^n$$

Worked Example 6

Laura invested $2500 for 5 years at an interest rate of 8% p.a. with interest compounding annually. Complete the table by calculating the values A, B, C, D, E and F.

Time period (n + 1)	**V_n ($)**	**Interest ($)**	**V_{n+1} ($)**
1	2500	**A**% of 2500 = 200	2700
2	**B**	8% of **C** = 216	**D**
3	2916	8% of 2916 = 233.28	3149.28
4	3149.28	8% of 3149.28 = 251.94	**E**
5	**F**	8% of 3401.22 = 272.10	3673.32

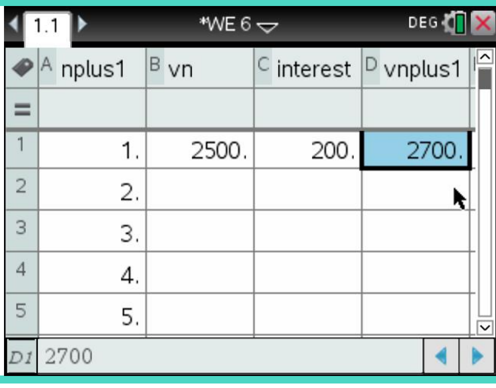
Worked Example 6 on CAS calculator

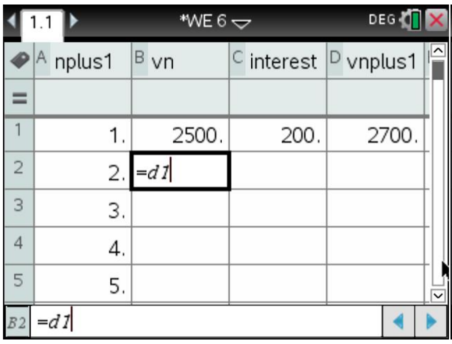
Enter the labels “n+1”, “V_n”, “Interest”, “V_{n+1}”

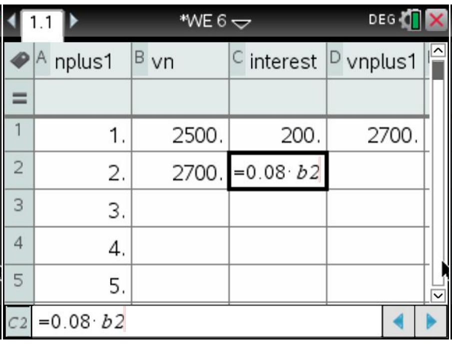
Note: You can’t use + on the CAS so spell it out

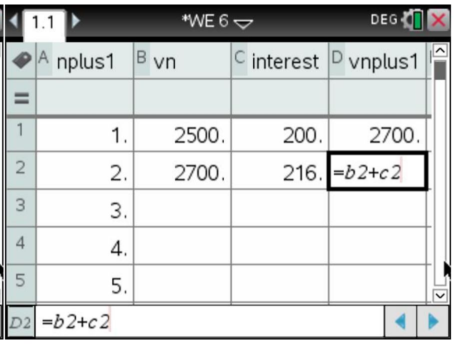
Next enter 1 to 5 in column A, and the starting values for V_n=2500, Interest=200 and V_{n+1}=2700 in cells b1, c1 and d1 respectively.

Then enter formulas shown below into cells b2, c2 and d2

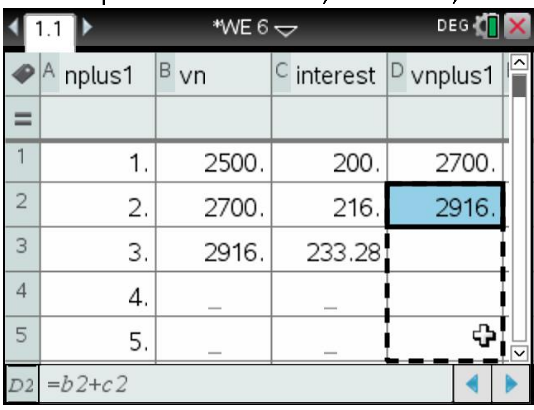


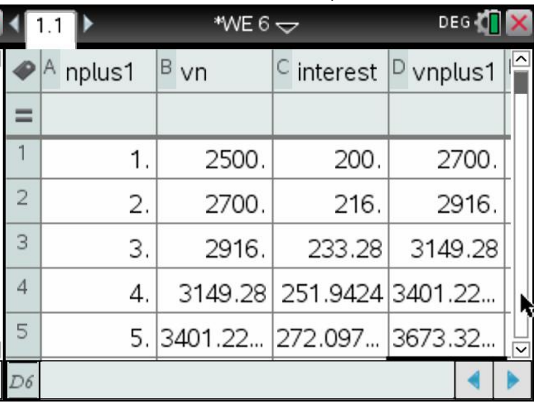






Now fill down the equations of cells b2, c2 and d2, downward for each of columns b, c and d.





The last screen picture shows the completed table.

6.4 Compound interest formula

From the previous section (6.3) we can see that we could write the value of the investment in terms of its previous value and hence, express it as the recurrence relation:

$$V_{n+1} = V_n R$$

where V_{n+1} is the amount of the investment 1 time period after V_n, R is the growth or compounding factor $\left(= 1 + \frac{r}{100}\right)$ and r is interest rate per period.

This pattern can be written in terms of the initial investment. This is the compound interest formula.

$V_n = V_0 R^n$ **where**

- V_n = **final or total amount ($)**
- V_0 = **principal ($)**
- R = **growth or compounding factor** $\left(= 1 + \frac{r}{100}\right)$
- r = **interest rate per period**
- n = **number of interest-bearing periods**

This formula gives the total amount in an account, not just the interest earned.

To find the total interest compound, *I*:

$I = V_n - V_0$ **where**

- V_n = **final or total amount ($)**
- V_0 = **principal ($)**

<u>Worked Example 7</u>
$5000 is invested for 4 years at 6.5% p.a., interest compound annually.
a) Generate the compound interest formula for this investment.

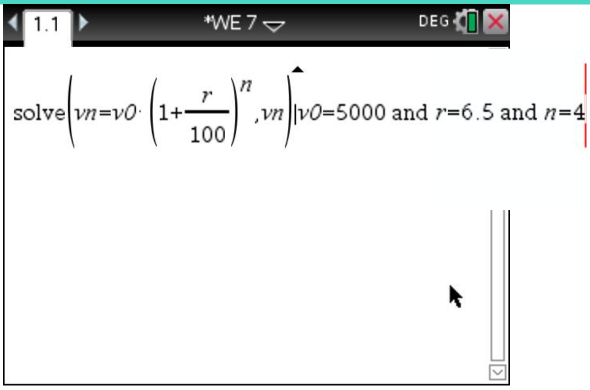
b) Find the amount in the balance after 4 years and the interest earned over this period.

Worked Example 7 on CAS calculator

On a calculator page
Using the Solve function
Enter the compound interest formula

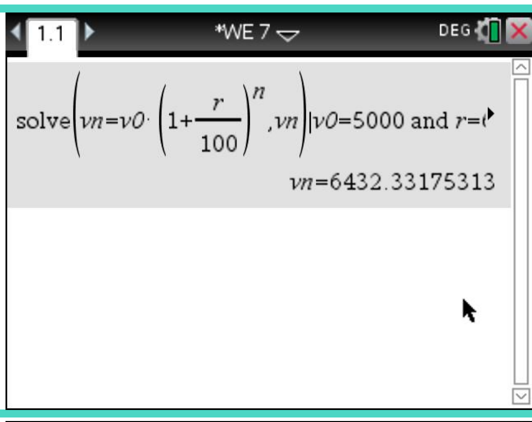
$$V_n = V_o\left(1+\frac{r}{100}\right)^n$$

and set the values of
V_0=$5000, r=6.5 and n=4
using "|*"



Top Tip: You could save this document on your CAS and just change the values

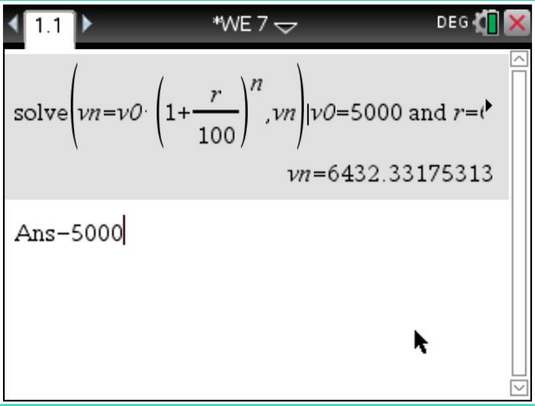
Press [enter] to get the value of V_n



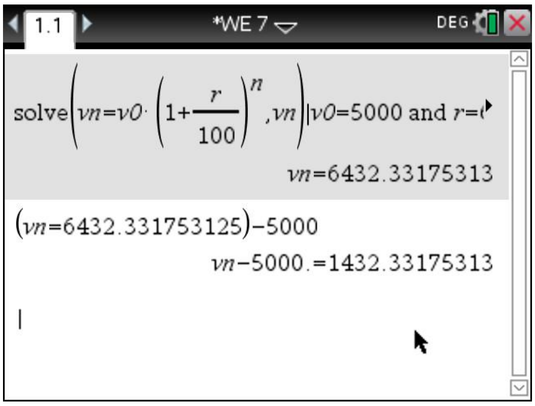
To find the interest earned, subtract the principal from the balance. (V_n-V_0)

On the CAS enter
[−][5][0][0][0]

(note: the CAS will insert ANS before the minus)



Press [enter] to get the answer



*| tells the CAS the values of variables, think of line entry as:
"**solve this**" (equation with variables) "**when**" the variables are…

Non-annual compounding

Many accounts can be compounded quarterly (every three months), weekly or daily. In these cases *n* and *r* are determined as follows:

$$\text{Number of interest periods, } n = \text{number of years} \times \text{number of interest periods per year}$$

$$\text{Interest rate per period, } r = \frac{\text{nominal interest rate per annum}}{\text{number of interest periods per year}}$$

Nominal interest rate per annum is the annual interest rate advertised by a financial institution.

Worked Example 8

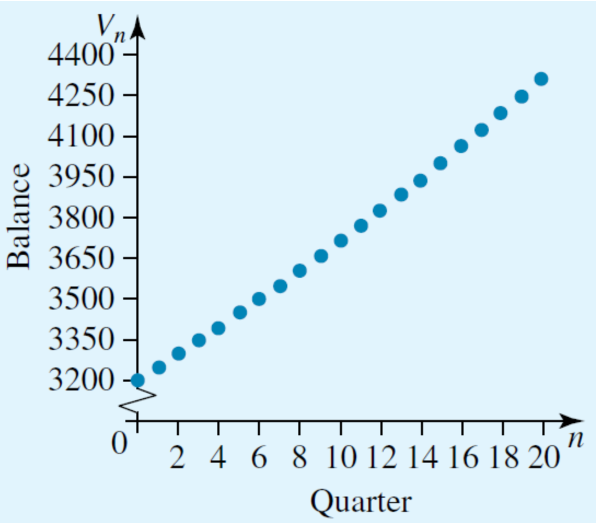
If $3200 is invested for 5 years at 6% p.a., interest compounded quarterly:

a) Find the number of interest bearing periods, *n*

b) find the interest rate per period, *r*

c) find the balance of the account after 5 years

d) graphically represent the balance at the end of each quarter for 5 years. Describe the shape of the graph.

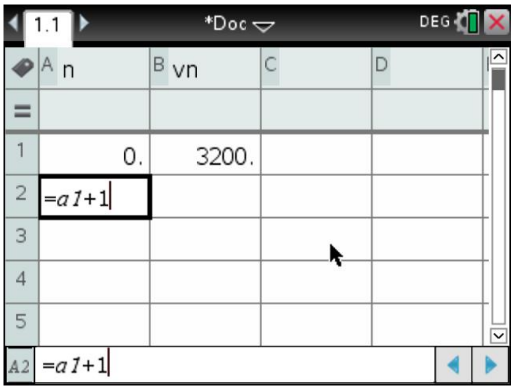


The graph is exponential as the interest is added at the end of each quarter and the following interest is calculated on the new balance.

Worked Example 8(c) and 8(d) on CAS calculator

Top Tip: Because we want to create a graph in 8(d) we will do this on a "list & spreadsheet" page

- On a "list & spreadsheet" page
- Label column A "n" and column B "Vn"
- In cell *a1* enter "0" and in *b1* enter the V_0 value of \$3200
- In cell *a2* enter the formula =*a1*+1, and then fill down (menu 3 3) to cell *a21* (from 8(a) n=20)

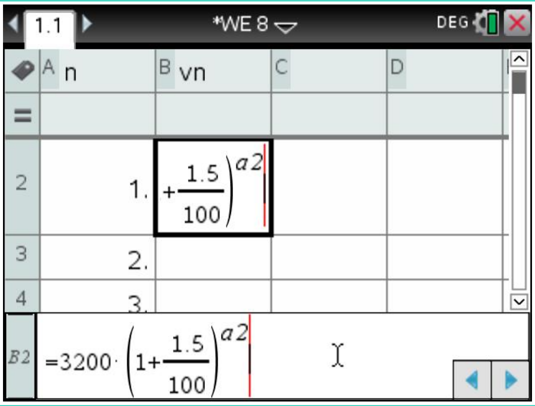


- In cell *b2* enter the formula

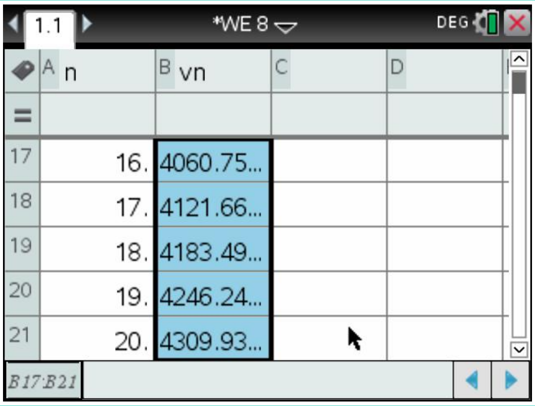
$$= 3200\left(1 + \frac{1.5}{100}\right)^{a2}$$

Note: r=1.5 is from part (b) $r = \frac{6}{4}$

- Press enter to get the value of V_1

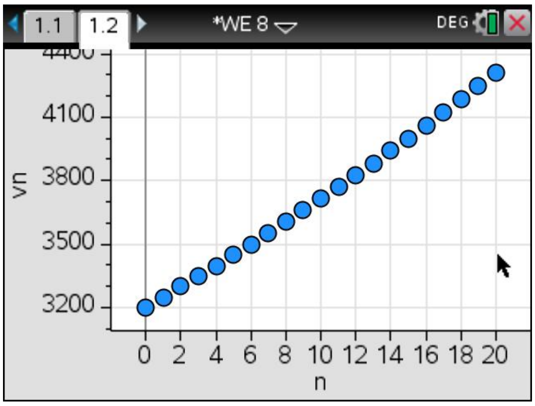


- Then fill down (menu 3 3)



- Add a "data & statistics" page (ctrl doc▾)

Label the x-axis "n"(Quarters) and the y-axis "Vn" (Balance)



Worked Example 9

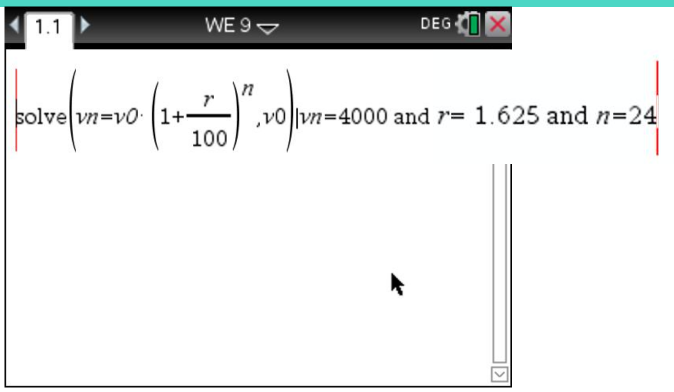
Find the principal that will grow to \$4000 in 6 years, if interest is added quarterly at 6.5% p.a.

Worked Example 9 on CAS calculator

On a calculator page
Using the Solve function
Enter the compound interest formula

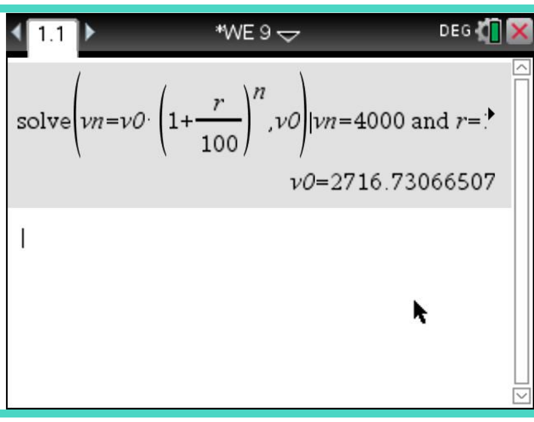
$$V_n = V_o\left(1 + \frac{r}{100}\right)^n$$

and set the values of
V_0=\$4000, r=1.625 and n=24 using "|"



Top Tip: You could save this document on your CAS and just change the values

Press [enter] to get the value of V_0



Modelling Geometric Growth and Decay

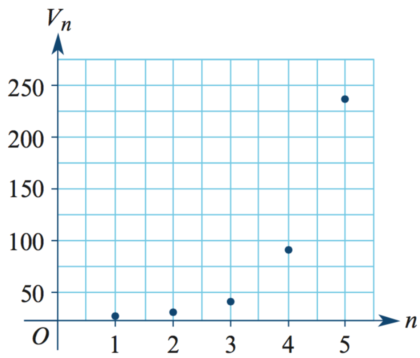
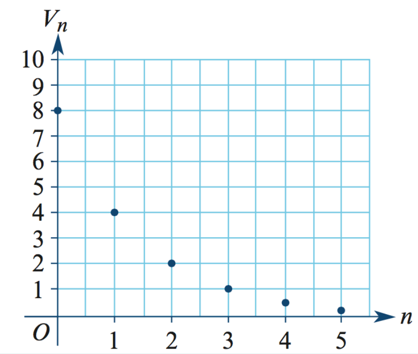
Compound interest is a geometric growth or decay. That is, it is a non-linear growth or decay. This is due to the fact that the *rule* governing compound interest is an increase (or decrease) by the same rate/Interest (as a percentage) at regular intervals.

Consider the following recurrence relations:

$$V_0 = 1, \quad V_{n+1} = 3V_n$$

$$V_0 = 8, \quad V_{n+1} = 0.5V_n$$

both have rules that generate a geometric pattern shown below

Recurrence relation	*Rule*	*Sequence*	*Graph*
$V_0 = 1$, $V_{n+1} = 3V_n$	'multiply by 3'	$1, 3, 9, \ldots$	
$V_0 = 8$, $V_{n+1} = 0.5V_n$	'multiply by 0.5'	$8, 4, 2, \ldots$	

The first generates a sequence whose terms grow geometrically and the second one decays geometrically.

In general, the rule:

$$\boldsymbol{V_{n+1} = RV_n}$$

- is geometric growth if $R>1$
- is geometric decay if $R<1$

where R is the growth or compounding factor $\left(= 1 + \frac{r}{100}\right)$ and r is the interest rate per period.

6.5 Finding rate or time for compound interest

Occasionally we know how much we can afford to invest, as well as the future amount that we require at the end of the investment. This allows us to determine the interest rate required to ensure we reach our target investment (savings) goal. With this information we can 'shop' around to find the best financial institution that will provide that interest rate.

We must first find the interest rate per period, *r*, and convert this to the corresponding nominal rate per annum. This and finding the time or number of periods is difficult.

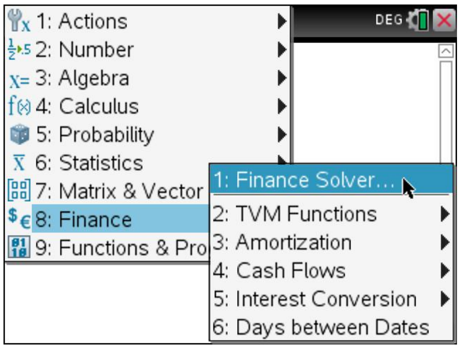
Your CAS has a finance function called **Finance Solver**. This can be used for compound interest calculations as shown in the worked examples in this section and in the future.

<u>Notes on the use of the Financial Solver</u>

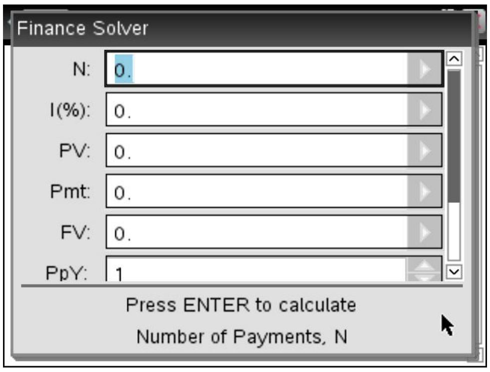
<u>Example of Finance Solver</u>
Find the amount of interest earned if **$3200** is invested for **5 years** at **6% p.a. compounded quarterly** using the Finance Solver.

On a calculator page

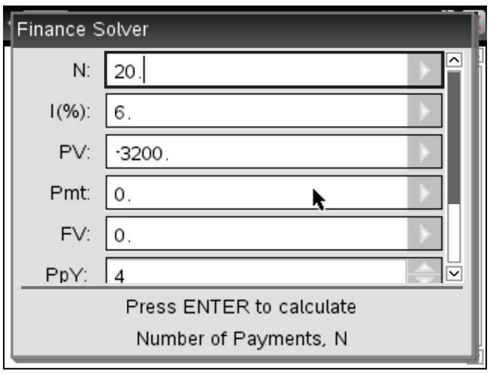
Press:
[menu] menu
[8] Finance
[1] Finance Solver



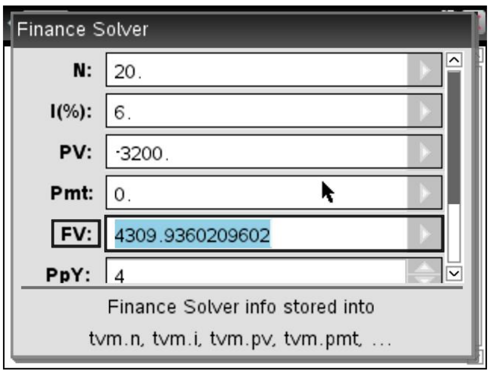
Complete the fields as shown.
N is the number of payments (20).
I(%) is the interest rate p.a. (6).
PV is the amount to be invested (-3200)*.
Pmt is the regular payment ($0).
FV is the future value of the investment (to be determined).
PpY is the number of payments per year (4).
CpY is the number of compounding period per year (4).
Press the tab key [tab] to move between fields.



Press the [tab] to return to the FV field and press [enter].



The investment is worth $4309.94 after 5 years.

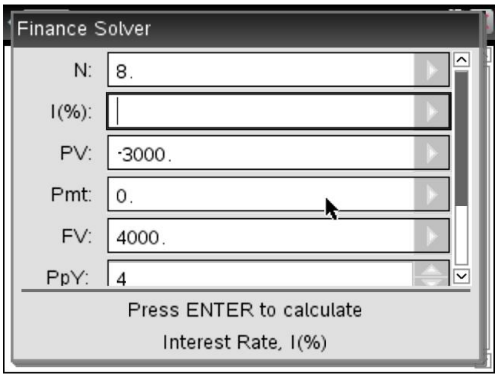


The interest earned is $4309.94 – $3200 = $1109.94.
*The principal value (PV) is entered as a negative value, because you give it to the bank. Hence the future value (FV) is a positive value to indicate it is given to you by the bank.

Worked Example 10

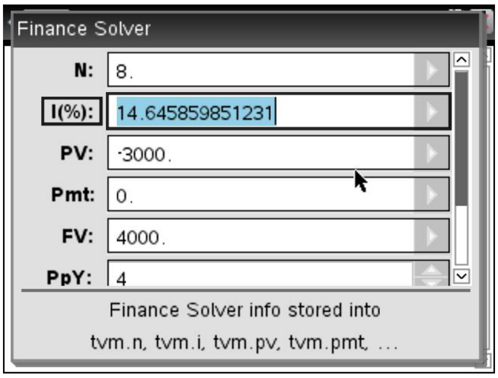
Find the interest rate per annum (correct to 2 decimal places) that would enable an investment of $3000 to grow to $4000 over 2 years if interest is compounded quarterly.

Complete the fields as shown.
N is the number of payments (8=2 years x 4 quarters).
I(%) is the interest rate p.a. (***to be determined – clear cell***).
PV is the amount to be invested (–3000) - ***you give money to bank***
Pmt is the regular payment ($0).
FV is the future value of the investment ($4000).
PpY is the number of payments per year (4).
CpY is the number of compounding period per year (4).
Press the tab key [tab] to move between fields.



Press the [tab] to return to the I% field and press [enter].

An annual Interest rate of 14.65% p.a. is required (correct to 2 decimal places).



Finding time in compound interest

To find *n*, the number of interest-bearing periods – the time period of an investment, we will use the Financial solver on the CAS.

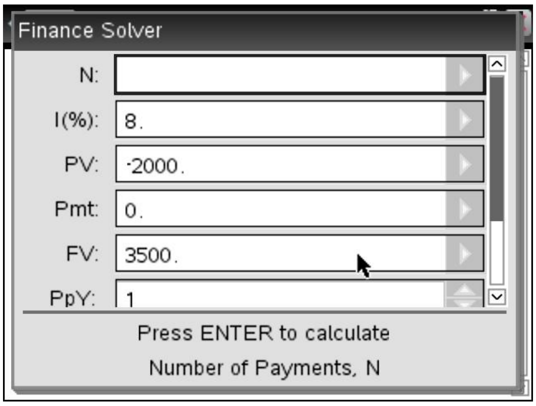
More often than not, the value obtained for *n*, the number will be a decimal, indicating the investment time is between two integers.

The smaller integer doesn't allow enough time for the investment to have the required balance and the larger integer represents more than the required time.

Worked Example 11

How long will it take $2000 to amount to $3500 at 8% p.a. with interest compounded annually?

Complete the fields as shown.
N is the number of payments (***to be determined – clear cell***)
I(%) is the interest rate p.a. (8%).
PV is the amount to be invested (–$2000) - ***money given away***
Pmt is the regular payment ($0).
FV is the future value of the investment ($3500).
PpY is the number of payments per year (1).
CpY is the number of compounding period per year (1).
Press the tab key [tab] to move between fields.

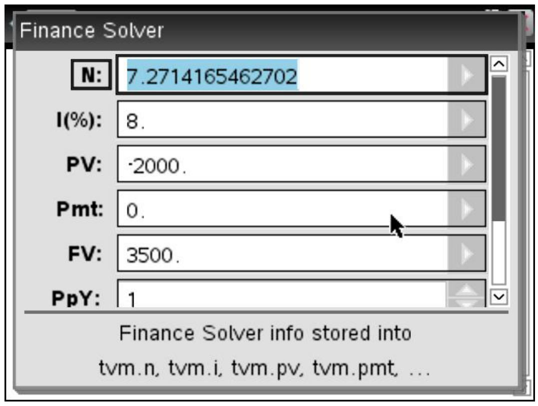


Press the tab to return to the N field and press enter.

As the Interest is compounded annually, so *n* represents years. Round *n* up to the next whole year.

Write your answer in words

“It will take 8 years for $2000 to increase to $3500.”

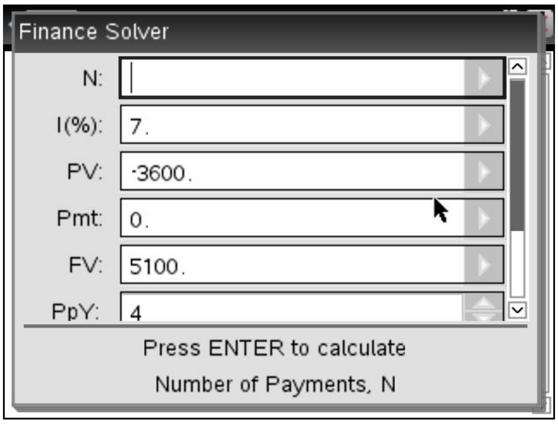


As discussed above, if we leave n=7.27years (or worse round it down to n=7 years) it won’t be long enough time for the investment to have reached the $3500 balance required. So, we often to round-up to the nearest whole number (n=8) because after 8 years sufficient interest periods (iterations) will have occurred to surpass the $3500 balance required.

<u>Worked Example 12</u>

Calculate the number of interest –bearing periods, *n*, required and hence the time it will take $3600 to amount to $5100 at a rate of 7 % p.a., with interest compounding quarterly.

Complete the fields as shown.
N is the number of payments (***to be determined - clear cell***).
I(%) is the interest rate p.a. (7%).
PV is the amount to be invested (–$3600).
Pmt is the regular payment ($0).
FV is the future value of the investment ($5100).
PpY is the number of payments per year (4).
CpY is the number of compounding period per year (4).
Press the tab key tab to move between fields.

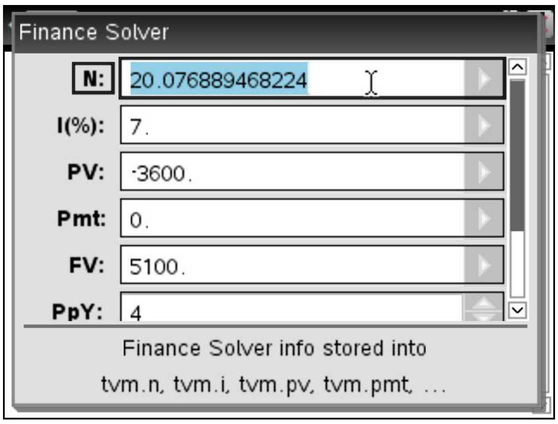


Press the tab to return to the N field and press enter.

As the Interest is compounded quarterly, so *n* represents quarters. Round *n* up to the next whole quarter. So, n=21 quarters

Write your answer in words

“It will take 21 quarters or 5 ¼ years for $3600 to increase to $5100.”



6.6 Flat rate depreciation

Some items such as antiques, jewellery and real estate increase in value (appreciate or increase in capital gain. Computers, vehicles or machinery decrease in value (depreciate) with time due to wear and tear, advances in technology or lack of demand.

Depreciation is the estimated loss in value of assets. The estimated value of an item at a point in time is called its **future value** (book value).

When the value becomes zero, the item is *written off*. At the end of an item's useful life its future value is called its **scrap value.**

> **Future value = cost price − total depreciation to that time**
> **When book value = \$0, then the item is said to be written off.**
> **Scrap value is the book value of an item at the end of its useful life.**

There are 3 methods in which to calculate depreciation:

1. flat rate depreciation
2. reducing balance depreciation
3. unit cost depreciation

Flat rate (straight line depreciation)

If an item depreciated by the **flat rate method,** then the value decreases by a fixed amount each time interval. It may be expressed in dollars or as a percentage of the original cost price.

As the depreciation value is the same for each interval, it is an example of straight line decay. This relationship can be expressed in the following recurrence relation:

$$V_{n+1} = V_n - d$$

where V_n is the value of the asset after n depreciating periods and d is the depreciation each time period.

The future value can also be calculated after *n* periods of depreciation.

$$V_n = V_0 - nd$$

We can use the above relationship or a depreciation schedule (table) to analyse flat rate depreciation.

Worked Example 13

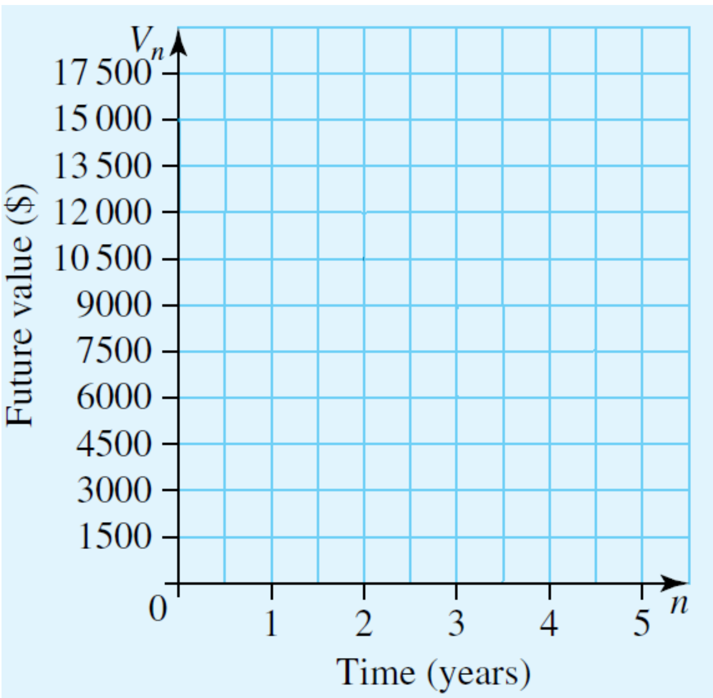
Fast Word Printing Company bought a new printing press for $15 000 and chose to depreciate it by the flat rate method. The depreciation was 15% of the prime cost each year and its useful life was 5 years.

a) Find the annual depreciation.

b) Set up a recurrence relation to represent the depreciation

c) Draw a depreciation schedule for the useful life of the press and use it to draw a graph of book value against time.

Time ***n* (years)**	**Depreciaton** ***d* ($)**	**Future value** **V_n ($)**
0		
1		
2		
3		
4		
5		

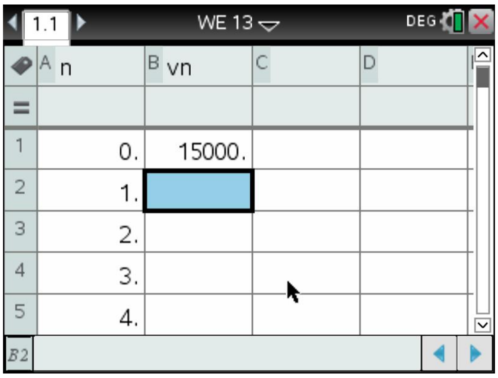


d) Generate the relationship between the book value and time and use it to find the scrap value.

Worked Example 13(c) and (d) on CAS calculator

13(c) On a lists & spreadsheet page

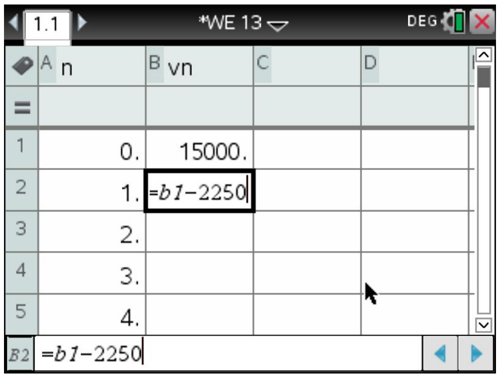
- Label column A "n" and column B "V_n"
- Enter 0 to 4 in the n column and the starting value 15000 (V_0) in cell b1.



In cell b2

- Enter the equation "=b1-2250"

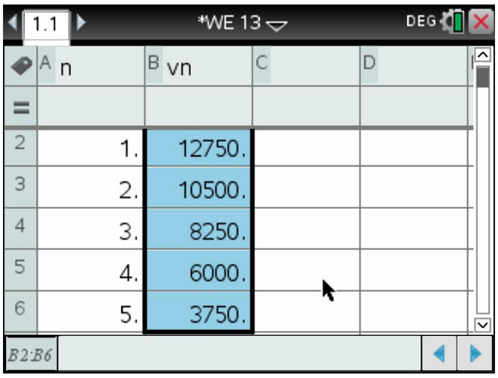
This equation is just $V_{n+1}=V_n-2250$ found in part (b)
Note: the 2250 is the **annual depreciation** found in part (a)



Press enter, then

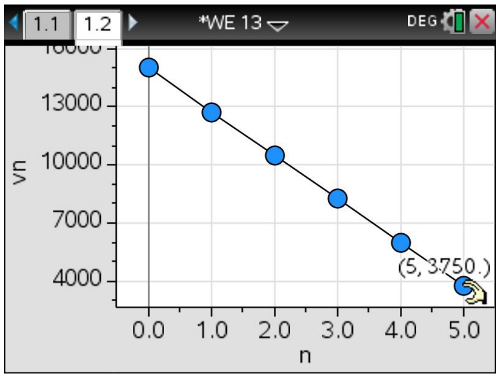
- fill down (menu 3 3) until n=5

V_n=3750 when n=5. So, this is the scrap value



Add a Data & Statistics page

- Label the x-axis "n" and the y-axis "V_n"



In this worked example the depreciation schedule gives the scrap value, when n=5 V_n=$3750. This can also be seen in the graph of book value against time, since it is only drawn for the item's useful life and its end point is the scrap value.

Businesses need to keep records of depreciation of all their assets on a year- to-year basis, for tax purposes.

What if you want to investigate the rate at which an item has depreciated over many years? A car, computer or mobile phone? If a straight line depreciation model is chosen, then the following example demonstrates its application.

<u>Worked Example 14</u>

Jarrod bought his car 5 years ago for \$15 000. Its current market value is \$7500. Assuming straight line depreciation, find:

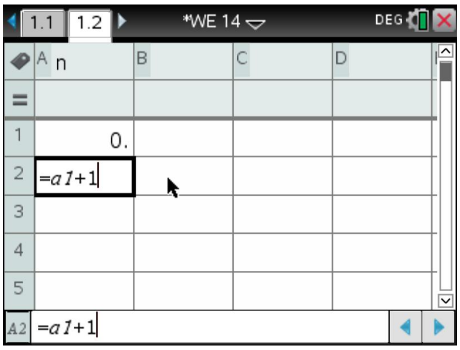
a) the car's annual depreciation rate

b) the relationship between the future value and time, and use it to find when the car will have a value of \$3000.

Worked Example 14 on CAS calculator

On a lists & spreadsheet page

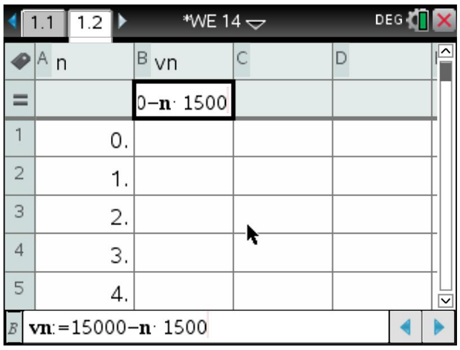
- Label column A "n" and enter 0 in cell a1, 1 in cell a2 etc, or in cell a2 enter "=a1+1 and **fill down** until n=10.



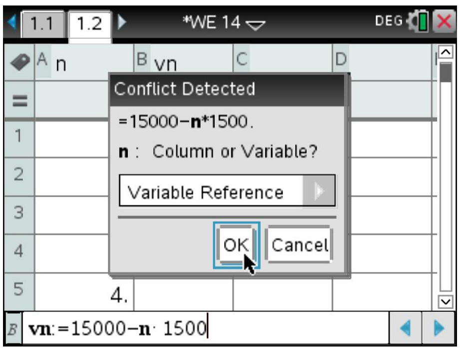
Label column B "V_n"In cell b2

- Enter the equation $= 15000 - n \times 1500$

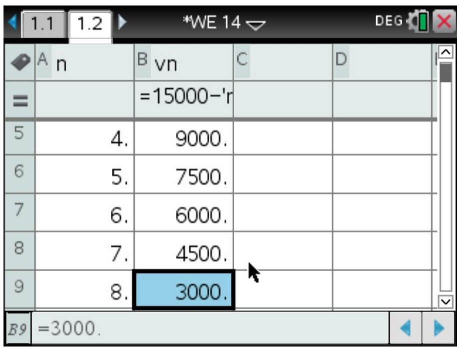
This equation is just $V_{n+1} = V_n - n \times d$, where d=1500 and V_0=15000



Press enter, the CAS needs to know if "**n**" is column n or a variable, IT IS A VARIABLE



- Click OK and the values for Vn will be shown
- Scroll down until it is 3000, and the value of n is 8



6.7 Reducing balance depreciation

If an item depreciates by the **reducing balance depreciation** method, its value reduces by a fixed value each time period. The rate is a percentage of the previous value of the item.

Reducing balance depreciation can be known as diminishing value depreciation.

Reducing balance depreciation can be expressed by the recurrence relation:

$$V_{n+1} = RV_n$$

where V_n is the value of the asset after n depreciating periods and $R = 1 - \frac{r}{100}$, where r is the depreciation rate.

<u>Worked Example 15</u>

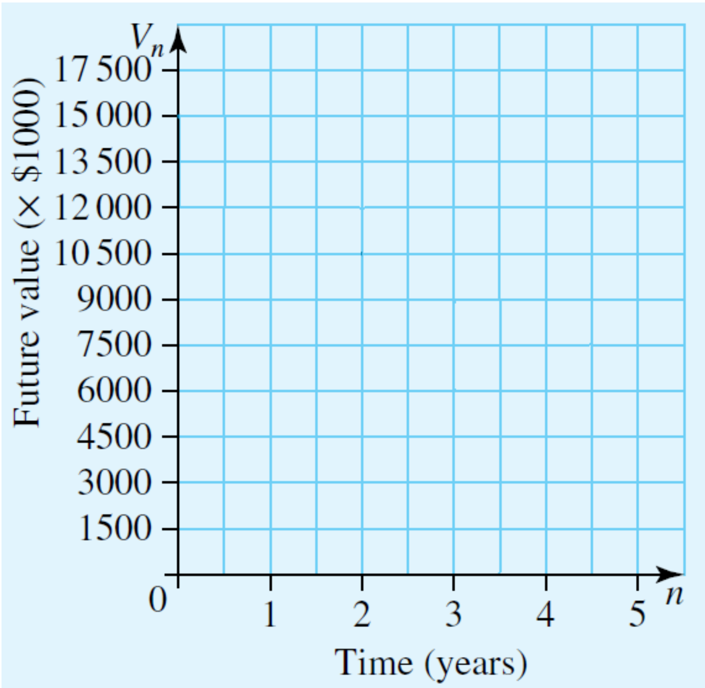
Suppose the new $15 000 printing press considered in Worked example 13 was depreciated by the reducing balance method at a rate of 20% p.a. of the previous value.

a) Generate a depreciation schedule using a recurrence relation for the first 5 years of work for the press.

Time *n* **(years)**	$V_{n+1} = RV_n$	**Future value** V_n **($)**
0	V_0 = 15000	
1	V_1 =	
2	V_2 =	
3	V_3 =	
4	V_4 =	
5	V_5 =	

b) What is the future value after 5 years?

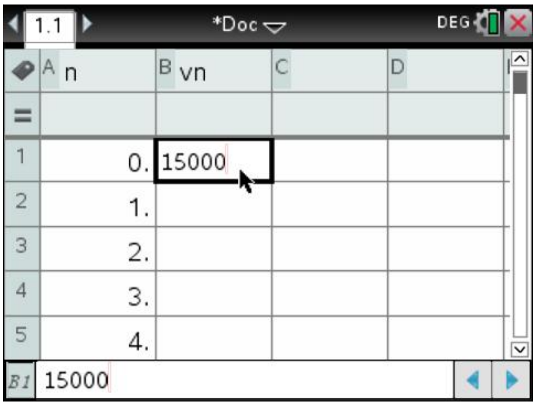
c) Draw a graph of future value against time.



Worked Example 15 on CAS calculator

On a lists & spreadsheet page

- Label column A "n" and column B "V_n"
- Enter 0 to 5 in the n column and the starting value 15000 (V_0) in cell b1.

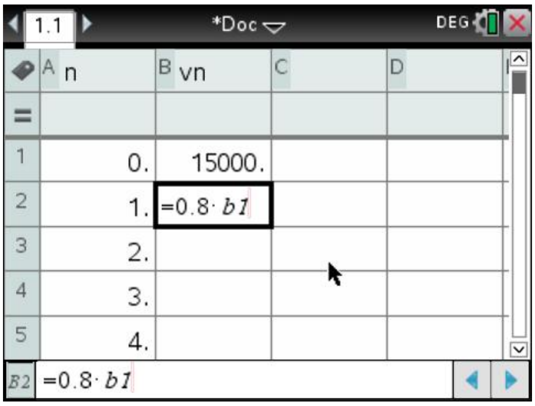


In cell b2

- Enter the equation "=0.8·*b1*"

Note: This equation is just $V_{n+1}=R\cdot V_n$

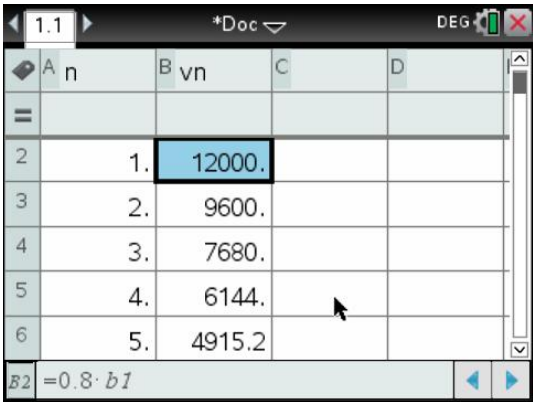
where R=0.8, $\left(R = 1 - \frac{r}{100}\right)$, and $r = 20\%$ p.a.



Press enter, then

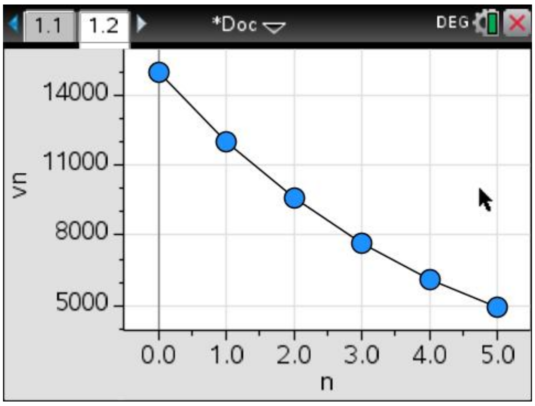
- fill down (menu 3 3) until n=5

V_n=$4915.20 when n=5. So, this is the value of the press after 5 years



Add a Data & Statistics page

- Label the x-axis "n" and the y-axis "V_n"



The Australian Tax Office (ATO) allows depreciation of an asset as a tax deduction, meaning that the depreciation reduces an individuals or businesses amount of tax to be paid. If using the reducing balance method, less tax will be paid at the beginning of the asset's life compared to the end of the asset's life, whereas a flat rate depreciation will have the same amount deducted for the asset's lifetime.

<u>A comparison between the two depreciation methods.</u>

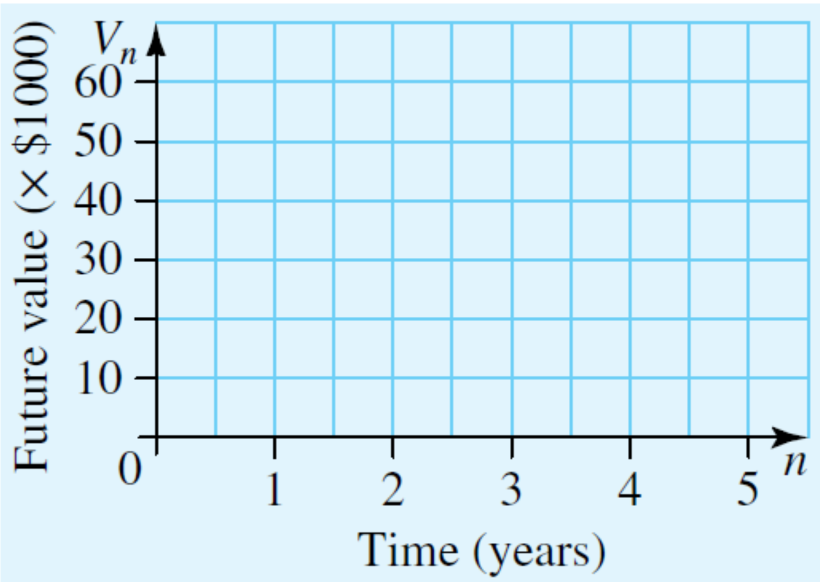
<u>Worked Example 16</u>

A transport business has bought a new bus for $60 000. The business has the choice of depreciating the bus by a flat rate of 20% of the cost price each year or by 30% of the previous value each year.

a) Generate depreciation schedules using both methods for a life of 5 years.

Flat rate			Reducing balance		
Time *n* **(years)**	**Depreciaton** *d* **($)**	**Future value** V_n **($)**	**Time** *n* **(years)**	$V_{n+1} = RV_n$	**Future value** V_n **($)**
0			**0**	V_0 =	
1			**1**	V_1 =	
2			**2**	V_2 =	
3			**3**	V_3 =	
4			**4**	V_4 =	
5			**5**	V_5 =	

b) Draw graphs of future value against time for both methods on the same set of axes.



c) After how many years does the reducing balance future value become greater that the flat rate future value?

____

____

____

____

Reducing balance depreciation formula

The reducing balance depreciation formula is:

$$V_n = V_0 R^n$$

V_n = book value after time, n

R = rate of depreciation $\left(= 1 - \frac{r}{100}\right)$

V_0 = cost price

n = time since purchase

Worked Example 17

The printing press from Worked example 13 was depreciated by the reducing balance method at 20% p.a. What will be the future value and total depreciation of the press after 5 years if it cost $15 000 new.

Effective life

We may know the scrap value of an item and we want to determine how long before the item reaches this value, i.e. it's useful or **effective life.**

In this case we use the reducing balance formula.

Worked Example 18

A photocopier purchased for $8000 depreciates by 25% p.a. by the reducing balance method. If the photocopier has a scrap value of $1200, how long will it be before this value is reached?

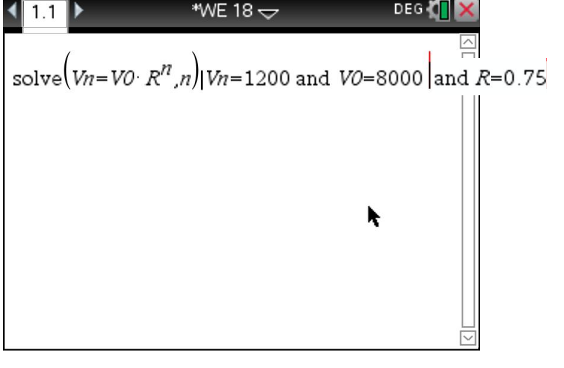
Worked Example 18 on CAS calculator

On a calculator page
Using the Solve function
Enter the reducing balance depreciation formula

$$V_n = V_o R^n$$

$$R = \left(1 - \frac{r}{100}\right)$$

and set the values of V_n=1200, V_0=8000 and R=0.75 using | symbol on the CAS



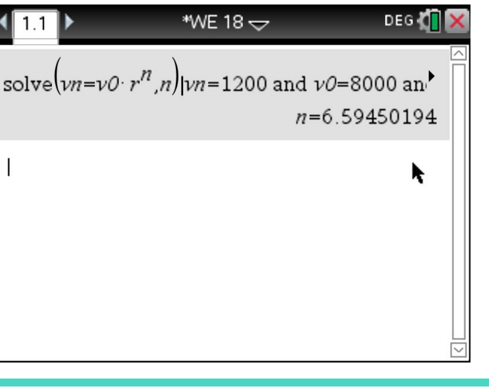
Top Tip: You could save this document on your CAS and just change the values

Press enter

The answers is n=6.5945

As the depreciation is calculated once a year, we need to round this up to n=7 years!

Answer: It will take 7 years for the photocopier to reach its scrap value



6.8 Unit cost depreciation

The **unit cost method** is based upon the maximum output (units) of the item. For example the useful life of a truck could be expressed in terms of the distance travelled rather than number of years. The actual depreciation per year would be a measure of the number of kilometres travelled.

Unit cost depreciation recurrence relation

The future value over time using unit cost depreciation can be expressed by the recurrence relation:

$$V_{n+1} = V_n - d$$

where V_n is the value of the asset after n outputs and d is the depreciation per output.

Worked Example 19

A motorbike purchased for $12 000 depreciates at a rate of $14 per 100 km driven.

a) Set up a recurrence relation to represent the depreciation.

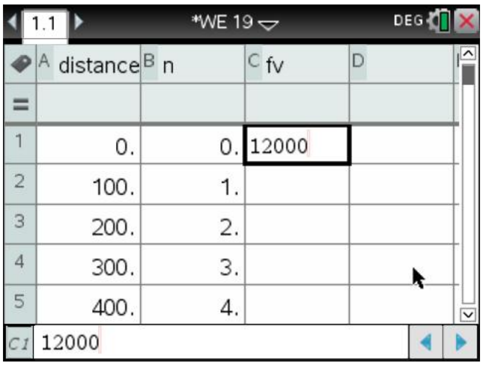
b) Use the recurrence relation to generate a depreciation schedule for the future value of the bike after it has been driven for 100 km, 200 km, 300 km, 400 km and 500 km.

Distance driven (km)	Outputs (n)	Future value V_n ($)
100		
200		
300		
400		
500		

Worked Example 19(b) on CAS calculator

On a lists & spreadsheet page

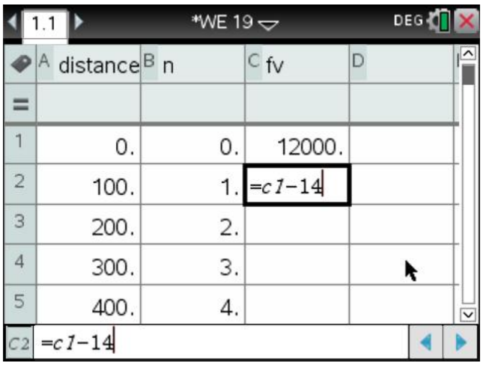
- Label column A "n" and column B "V_n"
- Enter 0 to 5 in the n column and the starting value 15000 (V_0) in cell b1.



In cell b2

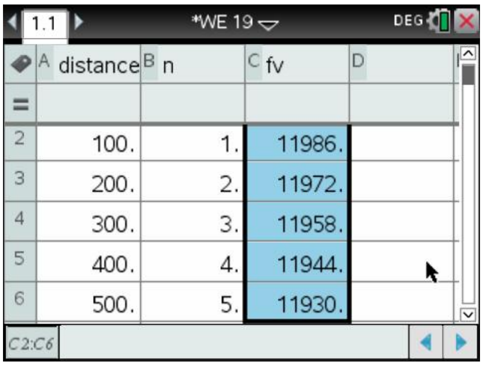
- Enter the equation "=c1–14"

Note: This equation is just $V_{n+1} = V_n - d$
Where V_n is the value of the asset after *n* outputs and *d* is the depreciation per output.



Press enter, then

- fill down (menu 3 3) until n=5



<u>Worked Example 20</u>

A taxi is bought for $31 000 and it depreciated by 28.4 cents per kilometre driven. In one year the car is driven 15 614 km. Find:

a) the annual depreciation for this particular year

b) its useful life if its scrap value is $12 000

Worked Example 21

A photocopier purchased for $10,800 depreciates at a rate of 20 cents for every 100 copies made. In its first year of use 500,000 copies were made and in its second year, 550,000. Find:

a) the depreciation each year

b) the future value at the end of the second year.

Unit cost depreciation equation

A future value after n outputs using unit cost depreciation can be expressed as:

$$V_n = V_0 - nd$$

where V_n is the value of the asset after n outputs and d is the depreciation per output.

If we were to use this equation with worked example 21

The rate d is $\frac{0.20}{100}$ (20 cents per 100 copies)

The number of copies n = 500,000+550,000=1,050,000

And V_0 = 10,800

$$V_n = 10800 - 1{,}050{,}000 \times \frac{0.20}{100}$$
$$V_n = 8{,}700$$

Worked Example 22

The initial cost of a vehicle was $27 850 and its scrap value is $5050. If the vehicle needs to be replaced after travelling 80 000 km (useful life):

a) find the depreciation rate (depreciation ($) per km)

b) find the amount of depreciation in a year when 16,497 km were travelled

c) set up an equation to determine the value of the car after travelling *n* km

d) find the future value after it has been used for a total of 60 000 km

e) set up a schedule table listing future value for every 20 000 km.

Use, *n* (km)	**Future value V_n ($)**
0	
20 000	
40 000	
60 000	
80 000	