

Year 11

General Mathematics 2017

Strand: Arithmetic and Number

Topic 3 – Financial Arithmetic

In this area of study students cover mental, by-hand and technology assisted computation with rational numbers, practical arithmetic and financial arithmetic, including estimation, order of magnitude and accuracy.

**This topic includes:**

• percentage increase and decrease applied to various financial contexts such as the price to earnings ratios of shares and percentage dividends, determining the impact of inflation on costs and the spending power of money over time, calculating percentage mark-ups and discounts, and calculating GST

• applications of simple interest and compound interest

• cash flow in common savings and credit accounts including interest calculation

• compound interest investments and loans

• comparison of purchase options including cash, credit and debit cards, personal loans, and time payments (hire purchase).

**Key knowledge**

• concepts of simple and compound interest and their application

• cash flow in common savings and credit accounts including interest calculations

• compound interest investments and debts.

**Key skills**

• apply ratio and proportion, and percentage and percentage change, to solve problems in a range of financial contexts

• apply simple interest to analyse cash flow in common savings and credit accounts

• apply compound interest to solve problems involving compound interest investments and loans

• compare the costs of a range of purchase options such as cash, credit and debit cards, personal loans, and time payments (hire purchase).

|  |  |
| --- | --- |
| **Chapter Sections** | **Questions to be completed** |
| **3.2** Percentage change | 1,2,3,4,5,6,7,9,16 |
| **3.4** Simple interest applications | 1,2,3,4,5,6,7,8,11,22 |
| **3.5** Compound interest applications | 1,2,3,4,5,6,7,8,9,11,22 |
| **3.6** Purchasing options | 1,2,3,4,5,6,7,8 |

# 3.1 Simple Interest and Compound Interest Tables

When you borrow money for a certain period of time from a bank or financial institution by taking a loan or mortgage, you must repay the original amount borrowed plus an extra amount called the ***interest***.

Similarly, if you lend money for a certain period of time to a bank or financial institution, you are expected to be rewarded by eventually getting your money back plus the interest.

There are two common ways that interests are calculated, they are known as ***Simple Interest*** and ***Compound Interest***.

## Simple Interest

If you invest or borrow money from the bank and you are reward or charged a *fixed* *amount* of interest *at regular time periods* it is called a **Simple Interest Loan.**

Simple Interest is an example of linear growth where the interest is usually calculated as a percentage of the amount invested or borrowed, the starting value (V0) or *Principal*. This constant amount is added at each payment over the duration of the loan.

Example: John invests $5 000 into a bank to save for his first car. The bank gives him 4 % interest per year. Using simple interest, calculate how much money will he have after 5 years if interest was to be calculated yearly?

Principal = Amount invested =

Interest rate =

Time/Period invested =

|  |  |  |  |
| --- | --- | --- | --- |
| **Time** | **Principal** | **Interest** | **Interest Accumulation** |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |

Total interest earnt after 5 years =

Total amount earnt =

Example: Kayla borrows $3 000 from a bank to put towards her savings for her oversea trip. The bank lends her for a period of 18 months with an interest rate of 7.8% pa. Using simple interest, how much does she owe after 18 months if the interest is calculated quarterly?

Principal = Amount borrowed =

Interest rate =

Time/Period borrowed =

|  |  |  |  |
| --- | --- | --- | --- |
| **Time** | **Principal** | **Interest** | **Interest Accumulation** |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |

Total interest owed =

Total amount of money Kayla owed =

The same information can be represented in the table below where Vn is the amount owe at nth period and Vn+1 is the amount owe in the next period.

|  |  |  |
| --- | --- | --- |
| **n + 1** | **Vn** | **Balance at the end of n + 1 time (Vn+1)** |
| 1 | V0 = | V1 = |
| 2 | V1 = | V2 = |
| 3 | V2 = | V3 = |
| 4 | V3 = | V4 = |
| 5 | V4 = | V5 = |
| 6 | V5 = | V6 = |

Represent the account balance for each of the 6 quarters graphically

**Exercise 3.1 (A)**:

1. For the following questions use table to calculate:
   1. the amount of simple interest, I, earned
   2. the total amount, A, at the end of the term.
      1. $1200 for 5 years at 10.5% p.a. calculated yearly.
      2. $8320 for 3 years at 6.45% p.a. calculated quarterly.
      3. $960 for years at 9.20% p.a. calculated monthly.
2. Calculate the simple interest that has to be paid if $4 650 is invested on term deposit for 7 years at 5.75% p.a.
3. Jenny wishes to invest $3 500 for 2 years at 4.95% p.a. but is not quite sure which option will give her the ultimate reward. Calculate the return for each of the investment and advice Jenny which option is best for her.
   * 1. annually
     2. semi-annually
     3. monthly

## Simple Interest using CAS calculator

Example: Jan invests $210 with building society in a fixed deposit account that paid 8% p.a. simple interest for 18 months.

a) How much did she receive after the 18 months?

|  |  |  |
| --- | --- | --- |
| Example on CAS Calculator | |  |
| Label column A “month”  Enter 0 in cell A1  In cell A2 enter: =a1+1  Fill down until the 18th month | Label column B “total”  Enter $210 in cell b1  In the next cell (B2) enter the equation  =210+1.4 x a2 | Now fill down this equation to the cells below.  Press  Menu b, data 3, fill 3 |

After 18 months of investment, Jane will receive

b) Represent the account balance for each of the 18 months graphically.

|  |  |
| --- | --- |
| Add a data and statistics page /~  Put the “month” on the x axis  and “total” on the y axis |  |

**Exercise 3.1 (B)**:

1. For the following questions use CAS calculator to calculate the total amount, A, owe at the end of the term when borrowing:
   1. $18 400 for 10 years at 6.89% p.a. calculated yearly.
   2. $2460 for 4 years at 5.75% p.a. calculated quarterly.
   3. $3730 for years at 7.39% p.a. calculated monthly.
   4. $126 000 for 3 years at 8.35% p.a. calculated daily.

## Compound interest

Unlike simple interest where the interest is the same for the duration of the loan, on the other hand, compound interest is calculated and added to the principal at the end of an interest-bearing period, and then both the principal and interest earn further interest during the next period, which in turn is added to the balance. This process continues for the life of the investment.

Both the balance of the account and interest increase at regular intervals.

Example: Consider $1000 invested for 4 years at an interest rate of 12% p.a. with interested compounded annually. What will be the final balance of the account?

|  |  |  |  |
| --- | --- | --- | --- |
| Time  (*n* + 1) | *Vn*($) | Interest ($) | Balance at the end of n +1 time  *Vn+1* ($) |
| 1 | V0 = |  | V1 = |
| 2 | V1 = |  | V2 = |
| 3 | V2 = |  | V3 = |
| 4 | V3 = |  | V4 = |
| 5 | V4 = |  | V5 = |

So the balance after 5 years is

Example5:

Laura invested $2500 for 5 years at an interest rate of 8% p.a. with interest compounding annually. Complete the table by calculating the values A, B, C, D, E and F.

|  |  |  |  |
| --- | --- | --- | --- |
| Time period  *(n* + 1) | *Vn* ($) | Interest ($) | *Vn+1* ($) |
| 1 | 2500 | **A**% of 2500 = 200 | 2700 |
| 2 | **B** | 8% of **C** = 216 | **D** |
| 3 | 2916 | 8% of 2916 = 233.28 | 3149.28 |
| 4 | 3149.28 | 8% of 3149.28 = 251.94 | **E** |
| 5 | **F** | 8% of 3401.22 = 272.10 | 3673.32 |

**Exercise 3.1 (C)**

1. Rosemary has $25 000 to invest for 2 years. She considers the following options:
   1. a term deposit at 6.75% p.a. compounded annually
   2. shares paying a dividend rate of 5.15% p.a. compounded quarterly
   3. a building society paying a return of 5.3% p.a. compounded monthly

## Compound Interest using CAS calculator

|  |  |
| --- | --- |
| Example on CAS calculator | |
| Enter the labels “n+1”, “Vn”, “Interest”, “Vn+1”  *Note: You can’t use + on the CAS so spell it out*  Next enter 1 to 5 in column A, and the starting values for Vn=2500, Interest=200 and Vn+1=2700 in cells b1, c1 and d1 respectively.  Then enter formulas shown below into cells b2, c2 and d2 |  |
|  | |
| Now fill down the equations of cells b2, c2 and d2, downward for each of columns b, c and d.    The last screen picture shows the completed table. | |

**Exercise 3.1 (D):**

1. **F**or the following questions use CAS calculator to calculate the total amount, A, earn at the end of the term when borrowing:
2. $18 400 for 10 years at 6.89% p.a. compounded yearly.
3. $2460 for 4 years at 5.75% p.a. compounded quarterly.
4. $3730 for years at 7.39% p.a. compounded monthly.

# 3.2 Percentage Change

Percentages can be used to give an indication of the amount of change that has taken place, which makes them very useful for comparison purposes. Percentages are frequently used in comments in the media. For example, a company might report that its profits have fallen by 6% over the previous year.

When we are using Percentage Change, we are referring to how much change has occurred compared to 100. This allows us to compare different changes.

Example: Which is the greater percentage change, 50 to 60 or 12 to 15?

To calculate Percentage Change find the change as a fraction then multiply by 100.

50 to 60 is a change of 10, so x 100 = 20%

12 to 15 is a change of 3, so x 100 = 25%

Therefore, relatively, 12 to 15 is a bigger change than 50 to 60.

Generally, the percentage change is found by taking the actual amount of change that has occurred and expressing it as a percentage of the starting value.

### Worked Example 1:

The price of petrol was $1.40 per litre but has now risen to $1.65 per litre. What is the percentage change in the price of petrol, correct to 2 decimal places?

### Calculating percentage change

#### To increase something by *x*% multiply by (100 + *x*)%

Example: An item is marked up by 3% of its cost price of $50. What is the marked up price?

The marked up price is: $50 + 3% of $50 =

Alternatively is to calculate: $50 × (100% + 3%) = $50 × 103% = $51.50

#### To reduce something by *x*% multiply by (100 − *x*)%

Example: An item is reduced its normal value by 3%. What is the sale price?

The sale price is $50 − 3% of $50 =

Alternatively is to calculate: $50 × (100% − 3%) = $50 × 97% = = $48.50

Worked Example 2: Increase $160 by 15%

# 3.3 Financial Applications of Ratios and Percentages

## Shares Dividends

Many people earn a second income through investments such as buying shares. A *dividend* is the payment of a company’s profit to its shareholders. To calculate a dividend, the profit shared is divided by the total number of shares in the company.

Example: If company Maths Co. makes a profit of $50 for the year and has 5 shares. The profit is divided equally by 5 shares, $50 ÷ 5 = $10, which is $10 is payed per share.

### Worked Example 3:

Calculate the dividend payable for a company with 2 500 000 shares when $525 000 of its annual profit is distributed to the shareholders?

## Percentage dividends

The *percentage dividend* allows us to compare how much different company’s shares are paying.

Maths Co. paid $10 for every share. If each share costs $200 to buy, to find the percentage dividend, find the dividend as a fraction then multiply by 100.

$10 from a $200 share is a dividend × 100 = 5%

Science Co. paid $20 for every share but each share costs $500 to buy.

$20 from a $500 share is a dividend × 100 = 4%

Therefore Maths Co. is paying a better return on your money.



### Worked Example 4:

Calculate the percentage dividend of a share that costs $13.45 with a dividend per share of $0.45. Give your answer correct to 2 decimal places.

## The price-to-earnings ratio

The price-to-earnings ratio (P/E ratio) is another way of comparing shares by looking at the current share price and the annual dividend.



### Worked Example 5:

Calculate the price-to-earnings ratio for a company whose current share price is $3.25 and has a dividend of 15 cents. Give your answer correct to 2 decimal places.

## Mark-ups and Discounts

#### Mark-ups are increases in prices

To find the mark-up on an item we multiply the item price by the percentage mark up.

Example: A new skirt was marked up 12% after the sale was over. It was $130.

Mark-up price = $130 + 12% of $130 = $130 + × $130 = $130 + $15.60 = $145.60

This can also be calculated as $130 x (100% + 12%) = $130 × 112% = $130 × = $145.60

#### Discounts are often expressed as a percentage

To find the percentage discount of an item we multiply the item price by the percentage discount.

Example: A new bike was on special for $500 with a further 8% off

Discount price = $500 – 8% of $500 = $500 − × $500 = $500 − $40 = $460

This can also be calculated as $500 x (100% – 8%) = $500 x 92% = $500 x = $460

### Worked Example 6:

A transport company adjusts their charges as the price of petrol changes. By what percentage, correct to 2 decimal places, do their fuel costs change if the price per litre of petrol increases from $1.36 to $1.42?

## Goods and Services Tax (GST)

In Australia we have a 10% tax that is charged on most purchases, known as a *goods and services tax* (or GST). Some essential items, such as medicine, education and certain types of food, are exempt from GST, but for all other goods GST is added to the cost of items bought or services paid for. If a price is quoted as being ‘inclusive of GST’, the amount of GST paid can be evaluated by dividing the price by 1.1, that is 100% + 10% = 110%.



**Amount of GST = Price with GST – Price without GST**

### Worked Example 7:

Calculate the amount of GST included in an item purchased for a total of $280.50.

## 3.4 Simple Interest Applications

Simple interest involves a calculation based on the original amount borrowed or invested; as a result, the amount of simple interest for the duration of particular loan is constant. For this reason simple interest is often called **flat rate** interest.

Example: John invests $5000 into a bank to save to buy his first car. The bank gives him with an interest rate of 4% per year. Using simple interest, how much money will he have after five years?

Principal = Amount invested = $5000

Interest rate = 7.8% p.a.

Time/Period invested = 5 years

|  |  |  |  |
| --- | --- | --- | --- |
| **Time** | **Principal** | **Interest** | **Interest Accumulation** |
| 1 | $5000 |  |  |
| 2 | $5000 |  |  |
| 3 | $5000 |  |  |
| 4 | $5000 |  |  |
| 5 | $5000 |  |  |

By observing how simple interest is calculated from the above table, we can see that under the Interest Accumulation column it follows a general rule:

Simple Interest = ***I*** =

Where ***I*** = the amount of interest earned or owe ($)

***P*** = Principal – amount of money invested or borrowed ($)

***r*** = Rate of the interest per year or per annum (pa) (%)

***T*** = Term - borrowed or investment period

### Worked Example 8:

Calculate the amount of simple interest earned on an investment of $4450 that returns 6.5% per annum for 3 years.

## Calculating the Principal, Rate or Time

The following formulas are derived from transposing the simple interest formula.

|  |  |  |
| --- | --- | --- |
|  |  |  |

### Worked Example 9:

How long will it take an investment of $2500 to earn $1100 with a simple interest rate of 5.5% pa?

## Simple Interest Loans

At the end of the borrowing or investing period, the total amount of money repay or reward is calculated as followed.

***A = P + I***

Where ***A*** = ***Total Amount*** repay or reward ($)

***P*** = Principal ($)

***I*** = Simple Interest ($)

### Worked Example 10:

Calculate the monthly payments for a $14 000 loan that is charged simple interest at a rate of 8.45% pa for 4 years.

# Cash flow

## Mon-annual interest calculations

Although interest rates on investments and loans are frequently quoted in terms of an annual rate, in reality calculations on interest rates are made more frequently throughout a year. Quarterly, monthly, weekly and even daily calculations are not uncommon.

### Worked Example 11:

How much interest is paid on a monthly balance of $665 with a simple interest rate of 7.2% pa?

## Minimum Balance Calculations

Banks and financial institutions need to make decisions about when to apply interest rate calculations on accounts of their customers. For investment accounts it is common practice to use the minimum balance in the account over a set period of time.

### Worked Example 12:

Interest on a savings account is earned at a simple rate of 7.5% p.a. and is calculated on the minimum monthly balance. How much interest is earned for the month of June if the opening balance is $1200 and the following transactions are made? Give your answer correct to the nearest cent.

|  |  |  |
| --- | --- | --- |
| **Date** | **Details** | **Amount** |
| June 2 | Deposit | $500 |
| June 4 | Withdraw | $150 |
| June 12 | Withdraw | $620 |
| June 18 | Deposit | $220 |
| June 22 | Withdraw | $500 |
| June 29 | Deposit | $120 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Date** | **Details** | **Withdraw** | **Deposit** | **Balance** |
| June 1 | Opening Balance |  |  |  |
| June 2 | Deposit |  | $500 |  |
| June 4 | Withdraw | $150 |  |  |
| June 12 | Withdraw | $620 |  |  |
| June 18 | Deposit |  | $220 |  |
| June 22 | Withdraw | $500 |  |  |
| June 29 | Deposit |  | $120 |  |

# 3.5 Compound Interest Applications

Simple interest rates calculate interest on the starting value. However, it is more common for interest to be calculated on the changing value throughout the time period of a loan or investment. This is known as compounding. In compounding, the interest is added to the balance, and then the next interest calculation is made on the new value.

Example: Consider an investment of $5000 that earns 5% p.a. compounding annually for 3 years.

|  |  |  |  |
| --- | --- | --- | --- |
| Time  (*n* + 1) | *Vn*($) | Interest ($) | Balance at the end of n +1 time  *Vn+1* ($) |
| 1 | V0 = $5000.00 |  | V1 = $5000.00 + $250.00 = $5250.00 |
| 2 | V1 = $5250.00 |  | V2 = $5250.00 + $262.50 = $5512.50 |
| 3 | V2 = $5512.50 |  | V3 = $5512.50 + $275.63 = $5788.13 |

Alternatively, the above table can be represented as

|  |  |  |
| --- | --- | --- |
| Time period | Balance at the end of n +1 time  *Vn+1* ($) | |
| 1 | V1 = |  |
| 2 | V2 = |  |
| 3 | V3 = |  |

#### Compounding Interest Formula

By observing how compound interest is calculated from the above table, the formula for Compound Interest is therefore:

*I = A - P*

Where **Vn** = ***A*** = the amount at the end of n compounding period ($)

***P*** = principal ($)

***r*** = rate of interest per period (%)

***n*** = number of compounding periods

### Worked Example 14:

Use the compound interest formula to calculate the amount of interest on an investment of $2500 at 3.5% p.a. compounded annually for 4 years, correct to the nearest cent.

#### Calculating the interest rate or principal

Transposing the compound interest formula gives the following formulas.

|  |  |
| --- | --- |
|  |  |

### Worked Example 15:

Use the compound interest formula to calculate the principal required, correct to the nearest cent, to have a final amount of $10 000 after compounding at a rate of 4.5% p.a. for 6 years.

## Non-annual compounding

Interest rates are usually expressed per annum (yearly), but compounding often takes place at more regular intervals, such as quarterly, monthly or weekly. When this happens, adjustments need to be made when applying the formula to ensure that the rate is expressed in the same period of time.

### Worked Example 16:

Use the compound interest formula to calculate the amount of interest accumulated on $1735 at 7.2% p.a. for 4 years if the compounding occurs monthly. Give your answer correct to the nearest cent.

## Inflation

Inflation is a term used to describe a general increase in prices over time that effectively decreases the purchasing power of a currency. Inflation can be measured by the inflation rate, which is an annual percentage change of the Consumer Price Index (CPI).

Inflation needs to be taken into account when analysing profits and losses over a period of time. It can be analysed by using the compound interest formula.

### Worked Example 17:

An investment property is purchased for $300 000 and is sold 3 years later for $320 000. If the average annual inflation is 2.5% p.a., has this been a profitable investment?

# 3.6 Purchasing Options

#### Cash Purchases

Buying goods with cash is the most straightforward purchase a person can make. The buyer owns the goods outright and no further payments are necessary. Some retailers or services offer a discount if a customer pays with cash.

### Worked Example 18:

A plumber offers a 5% discount if his customers pay with cash. How much would a customer be charged if they paid in cash and the fee before the discount was $139?

#### Credit cards

A credit card is an agreement between a financial institution (usually a bank) and an individual to loan an amount of money up to a pre-approved limit. Credit cards can be used to pay for transactions until the amount of debt on the credit card reaches the agreed limit of the credit card.

If a customer pays off the debt on their credit card within a set period of time after purchases are made, known as an interest-free period, they will pay no interest on the debt. Otherwise they will pay a high interest rate on the debt (usually 20–30% p.a.), with the interest calculated monthly. Customers are obliged to pay at least a minimum monthly amount off the debt, for example 3% of the balance.

Credit cards often charge an annual fee, but customers can also earn rewards from using credit cards, such as frequent flyer points for major airlines or discounts at certain retailers.

#### Debit cards

Debit cards are usually linked to bank accounts, although they can also be pre-loaded with set amounts of money. When a customer uses a debit card the money is debited directly from their bank account or from the pre-loaded amount.

If a customer tries to make a transaction with a debit card that exceeds the balance in their bank account, then either their account will become overdrawn (which typically incurs a fee from the banking facility), or the transaction will be declined.

### Worked Example 19:

Heather has a credit card that charges an interest rate of 19.79% p.a. She tries to ensure that she always pays off the full amount at the end of the interest-free period, but an expensive few months over the Christmas holidays leaves the outstanding balance on her card at $635, $427 and $155 for three consecutive months. Calculate the total amount of interest Heather has to pay over the three-month period. Give your answer correct to the nearest cent.

## Personal loans

A personal loan is a loan made by a lending institution to an individual. A personal loan will usually have a fixed interest rate attached to it, with the interest paid by the customer calculated on a reduced balance. This means that the interest for each period will be calculated on the amount still owing, rather than the original amount of the loan.

### Worked Example 20:

Frances takes out a loan of $3000 to help pay for a business management course. The loan has a fixed interest rate of 7.75% p.a. and Francis agrees to pay back $275 a month. Assuming that the interest is calculated before Francis’s payments, calculate the outstanding balance on the loan after Francis’s third payment. Give your answer correct to the nearest cent.

## Time payments (hire purchase)

A time payment, or hire purchase, can be used when a customer wants to make a large purchase but doesn’t have the means to pay up front. Time payments usually work by paying a small amount up front, and then paying weekly or monthly instalments.

## The effective rate of interest

It is often mistakenly believe that a loan is paid off basing on the interest rate advertised, what actually happen is the interest rate is usually slightly higher. This is because the interest is calculation is taken into account of the reducing balance of the amount owing after each payment has been made. This interest rate is known as the effective rate of interest.

Example: For a loan of $100 at 10% p.a. compounding quarterly over 2 years.

The effective annual interest rate:

This means that the effective annual interest rate is actually 10.38% and not 10%.

The comparison between the two can be shown in the following table.

|  |  |  |
| --- | --- | --- |
| Period | Amount owing ($) | Annual effective rate calculation ($) |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 (Year 1) |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 (Year 2) |  |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **Fortnightly** | **Monthly** | **Quarterly** | **Yearly** |
|  |  |  |  |  |
| 1 | V1 = $110.50 | V1 = $110.47 | V1 = $110.38 | V1 = $102.50 |
| 2 | V2 = $122.09 | V2 = $122.04 | V2 = $121.84 | V2 = $105.06 |
| 3 | V3 = $134.91 | V3 = $134.82 | V3 = $134.49 | V3 = $107.69 |
| 4 | V4 = $149.07 | V4 = $148.94 | V4 = $148.45 | V4 = $110.38 |
| Effective Annual Interest Rate | = 10.50% | = 10.47% | = 10.38% | = 10.00% |

The Effective Annual Interest Rate is the actual interest being calculated basing on the compounding term, that is if the compounding term is fortnightly, then the actual interest being calculated is 10.50%, not 10% as stated. Generally the Effective Annual Interest Rate will be slightly higher than the advertised annual interest figure when the compounding period is more frequent.

Generally, the effective rate of interest is used to give a more accurate picture of how much interest is actually charged at the time of payment.

An alternative formula can be used to calculate for the effective rate of interest



### Worked Example 21:

A furniture store offers its customers the option of purchasing a $2999 bed and mattress by paying $500 up front, followed by 12 monthly payments of $230.

1. How much does a customer pay in total if they choose the offered time payment plan?
2. What is the effective rate of interest for the time payment plan correct to 2 decimal places?