TSSM™
Creating VCE Success

FURTHER MATHEMATICS
Teach Yourself Series
Topic 6: Reducing balance loans and Annuities

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Reducing balance loans

In reducing balance loans, interest is usually charged every month by the financial institution and repayments are made by the borrower also on a regular basis. In such a situation, the rate at which the loan is paid off increases as the loan progresses.

Annuities as a recurrence relation

As it appears in Unit 3

- To find the amount still owing -

  - $V_{n+1} = R \times V_n - d$, given V_0

    where V_n = value of the investment after n time periods

    V_0 = initial (starting) amount

    d = payment each time period

    $R = 1 + \frac{r}{100}$, r = interest rate per period

Example. A loan of \$2400 is taken out with a reducing balance interest rate of 4.5% per annum with interest debited monthly. The borrower wishes to pay instalments of \$154.82 per month.

a. Write down a recurrence relation that will find the amount of loan left after n months.

$$V_{n+1} = 1.00375 \times V_n - 154.82, \quad V_0 = 2400$$

b. Use the recurrence relation to find the amount owing after 4 months.

$$V_0 = 2400$$
$$V_1 = 1.00375 \times 2400 - 154.82 = 2254.18$$
$$V_2 = 1.00375 \times 2254.18 - 154.82 = 2107.81$$
$$V_3 = 1.00375 \times 2107.81 - 154.82 = 1960.89$$
$$V_4 = 1.00375 \times 1960.89 - 154.82 = \$1813.43$$

c. What, to the nearest month, would be the term of such a loan?

Generate the table on CAS spreadsheet.

n = 16 instalments.

It will take 16 months to fully pay off the loan of \$2400.

Review Questions

1. Eddie takes a loan of $7200 at a rate of 12% p.a. (interest debited monthly) and is to be repaid with monthly instalments of $730.54.

a. Write down a recurrence relation to model this loan

b. Use the recurrence relation to find the how much Eddie will still owe after 6 months.

c. How much interest did Eddie pay in total after 10 months?

2. The term of a loan is 3 years during which instalments are paid fortnightly with interest debited fortnightly. The instalment with the greatest proportion being interest would be the:

A. 1^{st}
B. 92nd
C. 83^{rd}
D. 3rd
E. 35th

Using TVM solver

As it appears in Unit 4

- Used to find the number of repayments required to repay a loan in full

    - N = number of repayments
    - I% = nominal interest rate (% p.a.)
    - PV = amount borrowed/current amount owed (positive number)
    - PMT = regular payment amount (negative number)
    - FV = final amount owing (0 or negative number)
    - P/Y = number of payments per year
    - C/Y= number of compounds per year
    - PMT:END BEGIN – END should be left highlighted (charged every month)

Example. A charity organisation has $150 000 to set up a perpetuity as a grant used to fund their ongoing work. The charity invests in bonds that return 7.5% p.a. compounded annually. Use TVM Solver on a graphics calculator to calculate:

a. the amount of the annual grant

From the graphics calculator,
$Q = \$11\,250$

b. what interest rate (compounded annually) would be required if the perpetuity is to provide $12000 each year?

From the graphics calculator,
$r = 8\%$ p.a.

Review Questions

3. The private company "Lenders" offers loans of $30 000 to people at a rate of 6.5% p.a. (debited monthly). The loan is repaid in instalments of $504.30 over 6 years. The amount still owing after 3 years of repayments is:

A. $28967.50
B. $164316453.89
C. $8213.15
D. $30480.50
E. $36311.10

4. David has borrowed $45000 to buy a car. He agrees to repay the reducing balance loan over 15 years with monthly instalments at 9.3% p.a. (adjusted monthly).

a. Find the instalment value (using TVM solver).

b. The principal repaid during the 20^{th} repayment.

c. The interest paid during the 150^{th} repayment.

5. A loan of $11000 is being repaid by monthly instalments of $362.74 with interest being charged at 11.5% p.a. (debited monthly). Currently the amount owing is $7744.05.

a. How long will it take to reduce the amount outstanding to $2105.11?

b. How long will it take to repay the loan in full?

6. Jake wanted to borrow $42000 and was offered a reducing balance loan over 15 years at 9.75% p.a. (adjusted fortnightly)

a. Find the fortnightly repayment value.

b. What would be the term of the loan if the repayment was changed to $253.17?

c. What would be the term of the loan if the repayment was changed to $190.56?

7. A reducing balance loan of $80000 is taken out at 7.9% p.a. (adjusted monthly). If it is to repaid with monthly instalments of $639.84, the loan will be paid in full in

A. 10 years
B. 15 years
C. 20 years
D. 25 years
E. 22 years

8. If interest (r) of 7.5% p.a. is to be debited fortnightly on a loan requiring fortnightly repayments, then the growth rate (R) would be closest to:

A. 0.002
B. 0.003
C. 1.003
D. 1.002
E. 1.0075

9. If $1230 was borrowed at 6% p.a. by Jack, with instalments due monthly and interest debited monthly, then the required instalments needed to fully pay out the loan in 3 years would be:

A. $37.42
B. $17.25
C. $37.41
D. $17.42
E. none of the above

Reducing balance and flat rate comparisons

As it appears in Unit 4

- For flat rate loans – interest is calculated on the amount borrowed

    - $I = \frac{\text{Pr}T}{100}$

      where I = interest charged
      P = amount borrowed
      T = term of the loan
      r = interest rate per annum

- For reducing balance loans- interest is calculated on the amount outstanding each period (beneficial)

    - Interest charged = total repaid – amount borrowed

- Effective rate of interest – true indication of the interest rate on a loan

    - Estimation – little less than 2 × flat interest rate
    - Calculation - Effective interest rate $= \frac{2n}{n+1} \times$ flat rate (where n is the number of payments

Example. Adrian had $1245 that he had earned while working at a fast food establishment. He invested it in an interest bearing account for 36 months, earning 6.4% p.a. simple interest. How much interest should he have accrued?

$$I = \frac{PrT}{100}$$
$I = ?$ $P = \$1245$, $r = 6.4\%$,
$T = 36$ months
$= 3$ years
$$I = \frac{1245 \times 6.4 \times 3}{100}$$
$I = \$239.04$

Review Questions

10. Stephen wanted to borrow $24000 and was offered a reducing balance loan over 10 years at 6.6% p.a. (debited monthly)

a. Find the repayment value.

b. Find the total amount of interest paid.

c. Find the equivalent flat rate of interest for the loan if all the other variables are the same.

11. If the effective interest rate is 8.5% p.a. on a hire purchase with monthly repayments over 4 years, then the flat rate is closest to:

A. 16.7%
B. 17%
C. 4.3%
D. 9.7%
E. 4.1%

Perpetuities and annuity investments

As it appears in Unit 4

- Perpetuity – is an annuity where a permanently invested sum of money provides regular payments that continue forever

  ♦ $Q = \dfrac{\Pr}{100}$

  where Q = amount of the regular payment per period
  P = principal
  r = interest rate per period
  $R = 1 + \dfrac{r}{100}$ where r is the interest rate per payment period

- Annuity investment – is an investment where an initial sum and regular deposits are made (eg. Superannuation)

  ♦ $V_{n+1} = R \times V_n + d$

  where d = deposit each time period
  $R = 1 + \dfrac{r}{100}$ where r is the interest rate per payment period
  V_n = value of the investment after n payments

Example. An annuity investment has a current balance of $140 000 and is averaging an interest rate of 5% p.a. compounded monthly. The fund has another 20 years to go before the big payout. There is a monthly contribution of $900. Find, to the nearest thousand dollars, the final amount available at the end of the term.

P = 140 000, R =1.0042, n = 240, PMT = 900 I=5
Use TVM solver to get:
$749700

Review Questions

12. Keith is aged 45 and is planning to retire at 65 years of age. He estimates that he needs $480000 to provide for his retirement. His current superannuation fund has a balance of $60000 and is delivering 7% p.a. compounded monthly.

a. Find the monthly contributions needed to meet the retirement lump sum target.

b. If in the final 10 years before retirement, Keith doubles his monthly contribution calculated from **a**, find the new lump sum amount needed for retirement

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c. How much extra could Keith expect if the interest rate from part **b** is increased to 9% p.a. (for the final 10 years) compounded monthly.

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13. Adam has $20 000 to invest in an annuities investment. He contributes a further quarterly amount of $450 and earns an interest rate of 4% p.a., interest credited quarterly. Adam plans to withdraw the total amount in 5 years time. Which of the following is the final payout?

A. $26770.40
B. $23315.70
C. $34312.40
D. $73085.30
E. $14495.20

Amortisation table

As it appears in Unit 4

- An amortization table enables you to follow the repayment of a loan on a step-by-step basis.

Example. A student takes a loan personal loan for a new computer. She borrows $1600 at the rate of 12% p.a. compounded monthly. She will repay the loan with 6 equal monthly payments of $276.

Payment Number	Payment made	Interest paid	Principal reduction	Balance remaining
0	0	0	0	1600
1	276	16	260	1340
2	276	13.40	262.60	1077.40
3	276	10.77	265.23	812.17
4	276	8.12	267.88	544.30
5	276	5.44	270.56	273.74
6	276	2.74	273.26	0.48
Total	1656	56.47	1599.53	

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Solutions to Review Questions

1.

a. $V_{n+1} = 1.01 \times V_n - 730.54$, $V_0 = 7200$

b. Generate a table of values on CAS. $3148.65

c. $730.54 \times 10 + 310.215 - 7200 = \$415.62$

2. *Answer:* A

Explanation:

The first one will bear the greatest interest.

3. *Answer:* B

Explanation:

Use TVM Solver

4.

a. Use TVM solver
N = 180; I% = 9.3; PV = 45000; PMT = ?; FV = 0; P/Y = C/Y = 12
This gives PMT = -464.79 so instalment value is $464.79.

b. $134.03 (using the TVM solver)
Use the PMT value found in part a and find the difference between the amount owed (FV) after the 19^{th} payment (N = 19) and the amount owed after the 20^{th} payment (N = 20).

c. $98.85 (using the TVM solver)
Find the principal paid in the 150^{th} payment using the method in part b. Then subtract this from the instalment amount of $464.79 to find the interest paid in the 150^{th} repayment.

5.

a. n = N = ?

r = I% = 11.5

P_S = PV = 7744.05

Q = PMT = -362.74

A = FV = -2105.11

P/Y = 12

C/Y = 12

Put the above values on the TVM solver and it will give n = 18 months

b. Same as in part a with FV = 0 which gives n = 2 years

6.

a. $205.16 (Put the values in the TVM solver)

b. 10 years (Put the values in the TVM solver)

c. 18 years (Put the values in the TVM solver)

7. *Answer:* E

Explanation:

Put the values in the TVM solver.

8. *Answer:* C

Explanation:

$$1+\frac{7.5}{2400}=1.003$$

9. *Answer:* A

Explanation:

Use TVM solver to get $37.41898

10.

a. $273.74 (Put the values on the TVM solver)

b. $8848.80 (273.74 × 120 - 24000)

c. 3.69%

11. *Answer:* C

Explanation:

Use the formula-

$$\text{Effective interest rate} = \frac{2n}{n+1} \times \text{flat rate}$$

12.

a. Use TVM solver to get monthly repayment:
Q = $456.26

b. Use TVM solver to get the amount:
A = $558974.01

c. Difference = 665757.29 – 558974.01 = $106783.28

13. *Answer:* C

Explanation:

Substitute the values on TVM solver