5.4 Graphs of first-order recurrence relations

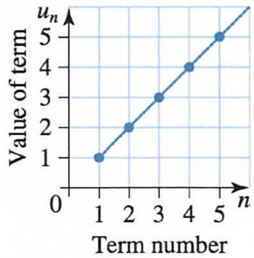
(Note: In this section the textbook uses n=1, 2, 3, … Instead of n=0, 1, 2, 3, … as they should have in line with the VCAA study design.)

In nature and business certain quantities may change in a uniform way. We can utilise graphs to represent changes and analyse the graphs, to find the next term.

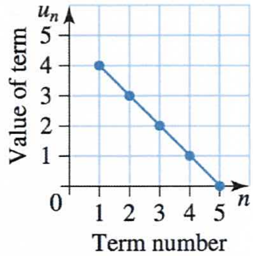
First-order recurrence relations: $u_{n+1} = u_n + b$ (arithmetic patterns)

b is a common difference.

The sequences of a first-order recurrence relation $u_{n+1} = u_n + b$ are distinguished by a straight line or a constant increase or decrease.



An increasing pattern or a positive common difference gives an upward straight line.



A decreasing pattern or a negative common difference gives a downward straight line.

Worked Example 8

On a graph, show the first five terms of the sequence described by the first-order recurrence relation:

$$u_{n+1} = u_n - 3 \qquad u_1 = -5$$

① Determine the first 5 terms when $n = 1, 2, 3, 4, 5$.

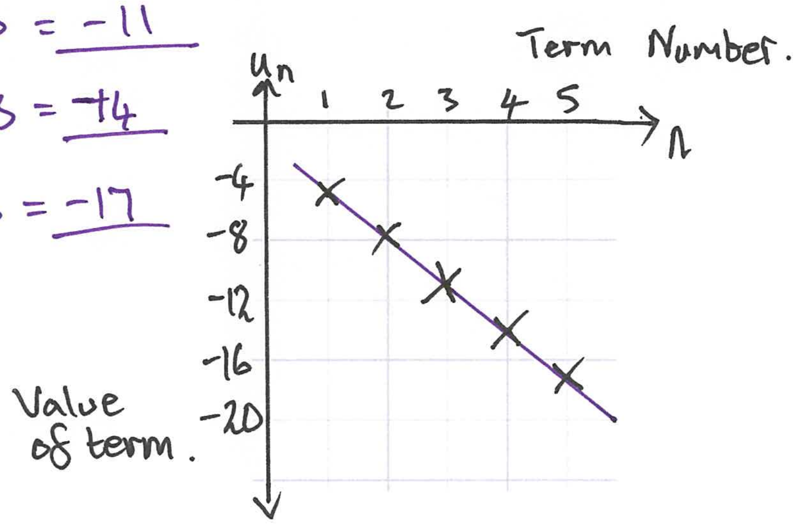
$u_1 = \underline{-5}$

$u_2 = u_1 - 3 = -5 - 3 = \underline{-8}$

$u_3 = u_2 - 3 = -8 - 3 = \underline{-11}$

$u_4 = u_3 - 3 = -11 - 3 = \underline{-14}$

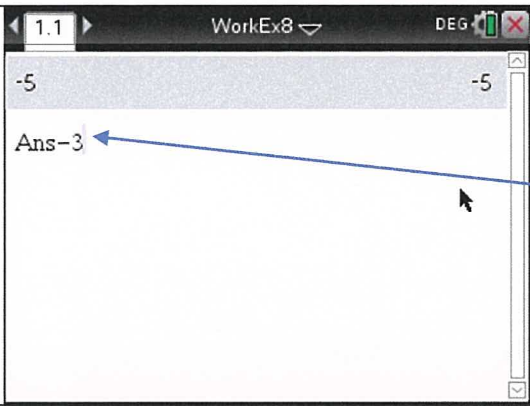
$u_5 = u_4 - 3 = -14 - 3 = \underline{-17}$



Worked Example 8: Using the CAS calculator

Enter the first term
- Enter -5 [(–)] [5]
- Press [enter]

Enter the equation
- Enter "Ans-3" or [ctrl] [(–)] [–] [3]



1[st] term

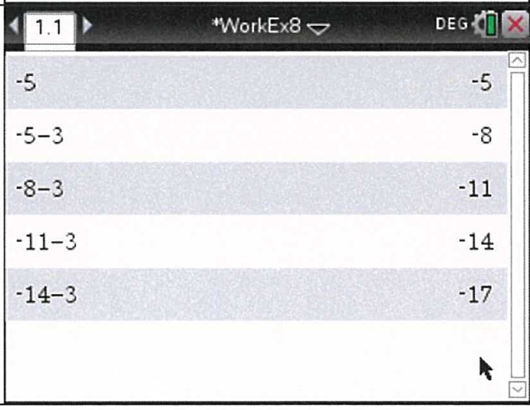
enter equation

Press [enter] for the 2[nd] term

Press [enter] for the 3[rd] term

Press [enter] for the 4[th] term

Press [enter] for the 5[th] term



1[st] term

2[nd] term

3[rd] term

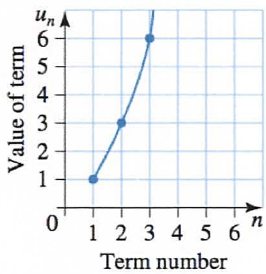
4[th] term

5[th] term

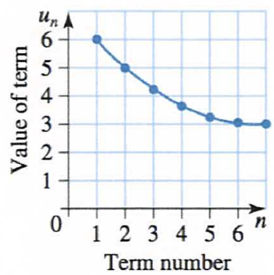
First-order recurrence relations: $u_{n+1} = Ru_n$ R = common ratio

The sequence of a first-order recurrence relation $u_{n+1} = Ru_n$ are distinguished by a curved line or a fluctuating (saw) form.

for example R = 1, 2, etc or 1.5, 5.5



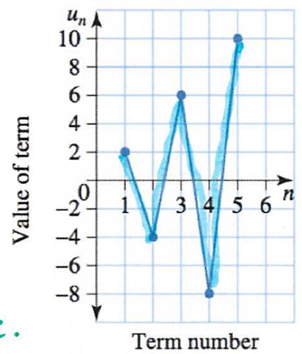
An increasing pattern or a positive common ratio greater than 1 ($R > 1$) gives an upward curved line.



A decreasing pattern or a positive fractional common ratio ($0 < R < 1$) gives a downward curved line.

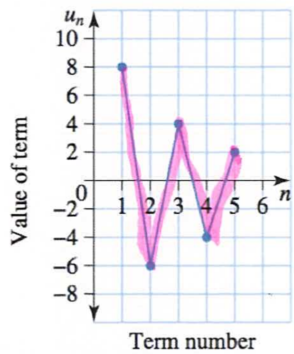
for example R = 0.5, 0.25 or $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ etc.

for example R = -2, -3 ... or -2.5, -3.75 etc.



further apart.

An increasing saw pattern occurs when the common ratio is a negative value less than –1 ($R < -1$).



getting closer.

A decreasing saw pattern occurs when the common ratio is a negative fraction ($-1 < R < 0$).

Worked example 9

On a graph, show the first five terms of the sequence described by the first-order recurrence relation:

$u_{n+1} = 4u_n \qquad u_1 = 0.5$

Determine the first 5 terms.

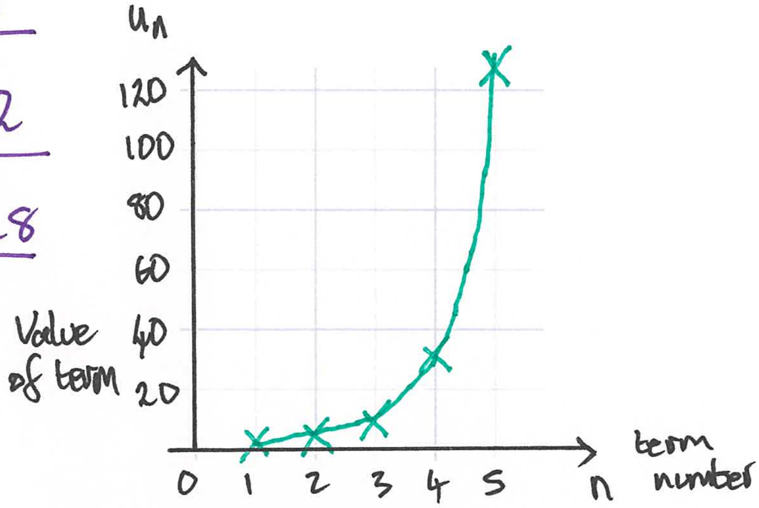
$\underline{u_1 = 0.5}$

$u_2 = 4 \times u_1 = 4 \times 0.5 \underline{= 2}$

$u_3 = 4 \times u_2 = 4 \times 2 \underline{= 8}$

$u_4 = 4 \times u_3 = 4 \times 8 \underline{= 32}$

$u_5 = 4 \times u_4 = 4 \times 32 \underline{= 128}$



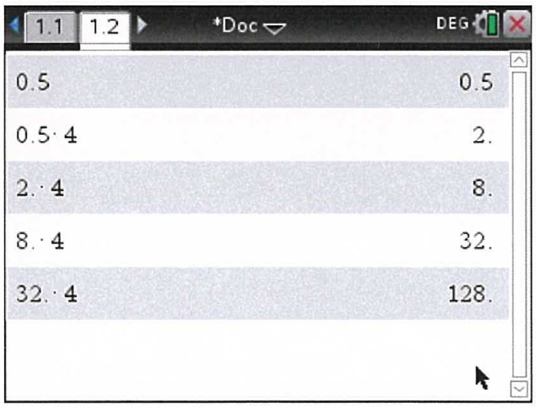
Worked example 9: Using CAS calculator

You can choose to determine the terms on the CAS and plot the graph by hand. Like this…

Enter the starting term:

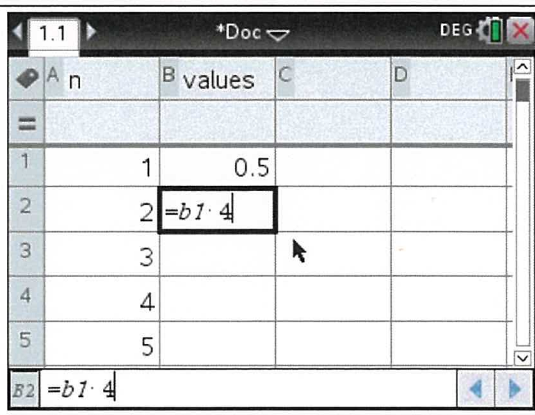
On a calculator page

- Enter the number [0][.][5][enter]
- Enter [ctrl][(–)][×][4][enter] for the 2nd term
- Press [enter] for the 3rd term
- Press [enter] for the 4th term
- Press [enter] for the 5th term



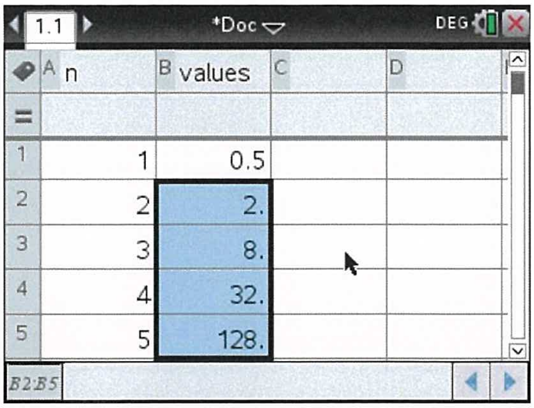
OR you could do it all on the CAS. Like this…

- Label column A "n"
- Enter the n values in column A
- Label column B "values"
- Enter the first term 0.5 in the first cell of column B
- In the next cell (B2) enter the equation after an =



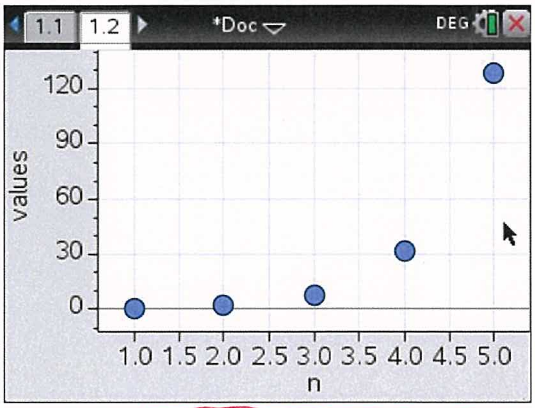
Now fill down this equation to the cells below.
Press

- Menu $\boxed{\text{menu}}$
- data $\boxed{3}$
- fill $\boxed{3}$



Add a data and statistics page $\boxed{\text{ctrl}}$ $\boxed{\text{doc}}$

Put the "n" on the x axis and "values" on the y axis



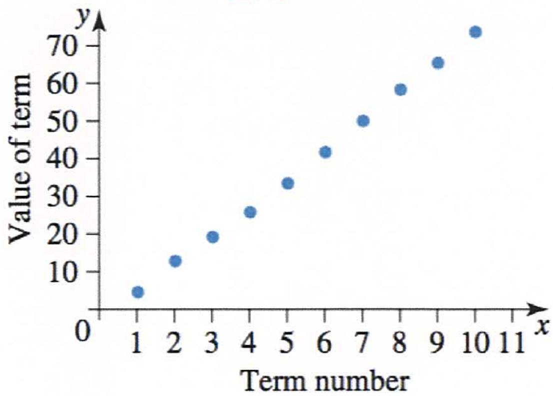
We have done, rule to graph. Now, graph to rule.

Interpretation of the graph of first-order recurrence relations

Is the graph Straight or Curved?

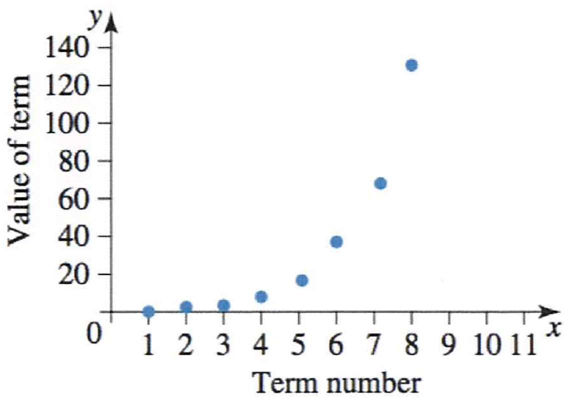
Straight or linear

> **A straight line or linear pattern is given by first-order recurrence relations of the form**
> $u_{n+1} = u_n + d$



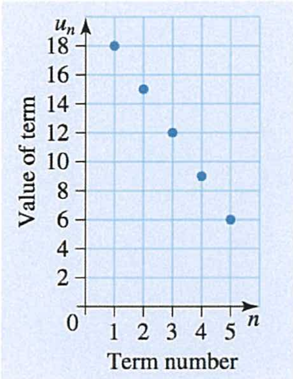
Non-linear (exponential)

> **A non-linear pattern is generated by first-order recurrence relations of the form** $u_{n+1} = Ru_n$



Worked Example 10

The first five terms of a sequence are plotted on the graph, Write the first-order recurrence relation that defines this sequence.



Step 1: Look for initial value.

In this case $n=1$ $u_1 = 18$

Step 2: Is it linear? (straight line)

Yes, it is linear.

So, it is of the form

$u_{n+1} = u_n + d$ (d is negative here)

Step 3: Determine d

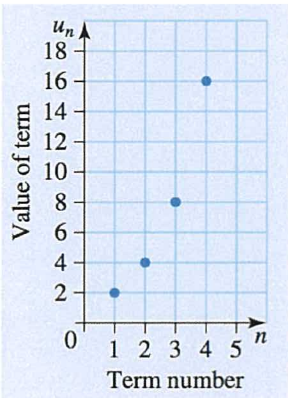
$u_2 - u_1 = 15-18 = -3$ $u_5 - u_4 = 6-9 = -3$

$u_3 - u_2 = 12-15 = -3$ so $d = -3$

$u_4 - u_3 = 9-12 = -3$ $\boxed{\text{Rule: } u_1 = 18 \quad u_{n+1} = u_n - 3}$

Worked Example 11

The first four terms of a sequence are plotted on the graph. Write the first-order recurrence relation that defines this sequence



The same steps as above.

1. $n=1$, $u_1 = 2$ 2. Not linear (check with ruler)

so it is $u_{n+1} = Ru_n$

3. Determine R (common ratio).

$\frac{u_2}{u_1} = \frac{4}{2} = 2$ $\frac{u_4}{u_3} = \frac{16}{8} = 2$

$\frac{u_3}{u_2} = \frac{8}{4} = 2$ $R = 2$

$$\boxed{u_1 = 2 \qquad u_{n+1} = 2u_n}$$

Worked Example 12

The first five terms of a sequence are plotted on the graph shown. Which of the following first-order recurrence relations could describe the sequence?

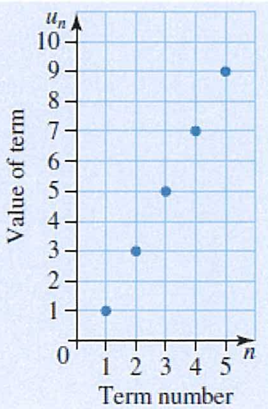
A $u_{n+1} = u_n + 1$ with $u_1 = 1$

B $u_{n+1} = u_n + 2$ with $u_1 = 1$

C $u_{n+1} = 2u_n$ with $u_1 = 1$

D $u_{n+1} = u_n + 1$ with $u_1 = 2$

E $u_{n+1} = u_n + 2$ with $u_1 = 2$



$u_1 = 1$

straight line

common difference is

$3-1 = 2$

$5-3 = 2$

$7-5 = 2$

$9-7 = 2$

so $u_{n+1} = u_n + 2$.

Answers B.

Graph Grids for chapter 5.4

Q	Q	Q
Q	Q	Q
Q	Q	Q
Q	Q	Q

Graph Grids for chapter 5.4

Q	Q	Q
Q	Q	Q
Q	Q	Q
Q	Q	Q

Student's Chapter 5 Summary Page