7.8 Annuity investments

A savings plan, like a Christmas Club account, where an initial principal is invested as well as regular deposits are made. The interest earned is calculated regularly on the balance of the investment, which increase with each regular deposit (annuity). This is similar to a reducing balance loan except the principal is growing and it is your own money, not a loan.

An annuity investment is an investment that has regular deposits made over a period of time.

$$V_{n+1} = V_n R + d$$
$$= V_n\left(1 + \frac{r}{100}\right) + d$$

Where: V_{n+1} = Amount after $n + 1$ payments
V_n = Amount at time n
r = Interest rate per period
d = Deposit amount

Compare this to the annuities formula on page 3 of these notes.

Here it is $+ d$

Worked Example 24

An initial deposit of \$1000 was made on an investment taken out over 5 years at a rate of 5.04% p.a. (interest calculated monthly), and an additional deposit of \$100 is made each month. Complete the table below for the first five deposits and calculate how much interest had been earned over this time.

$n + 1$	V_n	d	V_{n+1}
1	V_0=\$1000	\$100	V_1 = \$1104.20
2	V_1 = \$1104.20	\$100	V_2 = \$1208.84
3	V_2 = \$1208.84	\$100	V_3 = \$1313.92
4	V_3 = \$1313.92	\$100	V_4 = \$1419.44
5	V_4 = \$1419.44	\$100	V_5 = \$1525.40

$V_0 = \$1000$, $d = \$100$, $r = \frac{5.04}{12} = 0.42$

Calculate $V_1 = V_0 \times \left(1 + \frac{0.42}{100}\right) + 100 = \$1104.20$

Calculate $V_2 = V_1 \times \left(1 + \frac{0.42}{100}\right) + 100 = \$1208.84$

repeat for V_3, V_4 and V_5

Superannuation

By the time we retire, we will have to provide enough money to live on, in Australia this is called Superannuation. When we begin working, our employers are required by law to contribute money to a Superannuation fund, employees can also 'top up' their superannuation account by contributing money themselves. The money builds up over many years and can be withdrawn when a person reaches retirement age. The fund then can be placed into an annuity or perpetuity that pays for the retiree's living expenses and lifestyle.

It is important to determine how much money you will need to retire on, to have the lifestyle you would like, seeing a financial advisor can help.

The money in the Superannuation fund is invested in shares and property by superannuation fund managers. The performance of the funds varies year to year. For simplicity in determining the return of a superannuation fund, it will be assumed that the interest rate remains the same and outside influences such as taxation and inflation, will not be considered.

The money that builds up in these annuities investments can be calculated using the annuities formula, except the amount V_n grows with the addition of regular payments d.

$$V_n = V_0 R^n + \frac{d(R^n - 1)}{R - 1}$$

where:

V_0 = the initial amount invested

R = the compounding or growth factor

$= 1 + \frac{r}{100}$ (r = the interest rate per payment period)

d = the amount of the regular deposits made per period

n = the number of deposits

V_n = the balance after n deposits.

Compare this to the Annuities formula on page 5 of these notes!

Finance Solver can also be used in a similar way to reducing balance loans, with the difference of the cash flows are reversed (opposite sign). ← See example below.

Worked Example 25

Helen currently has \$2000 in a savings account that is averaging an interest rate of 8% p.a. compounding annually. She wants to calculate the amount that she will receive in 5 years when she plans to go on an overseas trip.

a) If she deposits \$6000 each year find (correct to the nearest \$1000) the amount available for her overseas trip.

Manually, $V_0 = \$2000$, $d = \$6000$, $n = 5$, $r = 8$

Calculate $R = 1 + \frac{8}{100} = 1.08$

Annuities Formula is $V_n = V_0 R^n + \frac{d(R^n - 1)}{R - 1}$.

$V_5 = 2000 \times (1.08)^5 + \frac{6000((1.08)^5 - 1)}{1.08 - 1}$

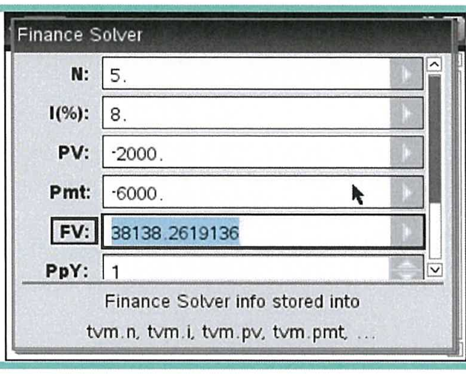
$= \$38{,}138.26$ is balance after 5 years.

Alternatively, use the Finance Solver

Using the Financial Solver, Enter the following:

n (N:) = 5
r (I(%)): = 8
P (PV:) = -2000 ← money in bank.
Pmt: = -6000 ← yearly deposits.
FV: = ?
PpY: = 1
CpY: = 1

Place the cursor on FV: Press ENTER to solve.



The final balance of the investment after 5 years is \$38,000. (to the nearest \$1000).

b) If she places her \$2000 and increases her deposits to \$7000 each year into a different savings account that can offer 9% p.a. compounding annually, find (correct to the nearest \$1000) the amount available for her overseas trip.

Now $V_0 = 2000$, $d = \$7000$, $n = 5$ and $r = 9\%$, $R = 1 + \frac{9}{100} = 1.09$

Annuities formula: $V_5 = 2000 \times (1.09)^5 + \frac{7000\left((1.09)^5 - 1\right)}{1.09 - 1}$

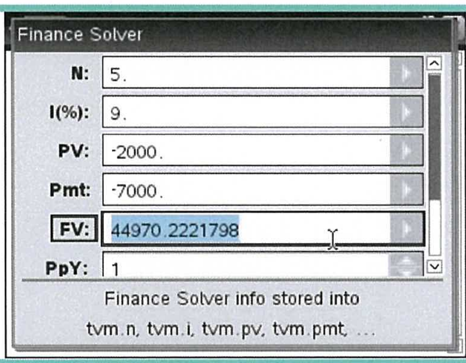
$= \$44970.22$ or \$45,000 (nearest \$1000).

Alternatively, use the Finance Solver

Using the Financial Solver, Enter the following:

n (N:) = 5
r (I(%)): = 9
P (PV:) = -2000
Pmt: = -7000
FV: = ?
PpY: = 1
CpY: = 1

Place the cursor on FV: Press ENTER to solve.



The final balance of the investment after 5 years is \$45000 (to the nearest \$1000).

c) Calculate the extra amount saved by investing \$7000 each year at 9% p.a. compared with \$6000 each year at 8% p.a.

for part (a) $V_5 = \$38138.26$

for part (b) $V_5 = \$44970.22$.

so extra $= \$44970.22 - \$38138.26$
$= \$6,831.96$

or \$7000 (to nearest \$1000).

Planning for retirement

When do you want to retire? How much money do you need? Are you planning to travel? As the life expectancy for Australian's is expected to be more than 80 years old, people will need to plan for 20 plus years of retirement. A common target is to have 60 to 65% of your pre-retirement income. So if you earn $60 000 p.a. now, your retirement income would be $36 000 p.a. (in today's dollars).

Planning for retirement should be regularly revisited, to ensure enough money is being invested. Financial planners are able to assist in this planning.

Worked Example 26

Andrew is aged 45 and is planning to retire at 65 years of age. He estimates that he needs $480 000 to provide for his retirement. His current superannuation fund has a balance of $60 000 and is delivering 7% p.a. compounded monthly.

a) Find the monthly contributions needed to meet the retirement lump sum target.

Note: $R = 1 + \frac{7/12}{100} = \frac{1207}{1200}$ or $1.0058\dot{3}$

PV = −60000 (in the bank).

r = I = 7%

CpY = 12

FV = 480000

n = 20 × 12 = 240.

Find monthly payments needed so PpY = 12 and pmt = ?

In this question we need to find d in the annuities formula using CAS solve function gives.

pmt = $ 456.26

Alternatively, use the Finance Solver

Using the Financial Solver, Enter the following:

n (N:) = 240   12 × 20

r (I(%): = 7

P (PV:) = −60 000

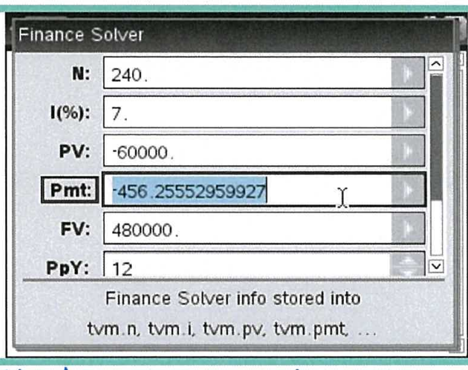
Pmt: = ______

FV: = 480,000

PpY: = 12

CpY: = 12

Place the cursor on Pmt: Press ENTER to solve.



Write Answer: The monthly contribution to achieve a retirement lump sum of $480000 is $456.26

b) If in the final ten years before retirement, Andrew doubles his monthly contribution calculated from **(a)**, find the new lump sum amount available for retirement.

Because Andrew changes his monthly contributions after 10 years. Here we need to calculate the FV (V_{120}) after 10 years and use this as the PV (V_0) for the next 10 years.

1st 10 years $V_{120} = 60000 \times R^{120} + \frac{d(R^{120}-1)}{R-1} = \$199551.36$

Next 10 years $V_{120} = \$199551.36 \times R^{12} + \frac{912.52(R^{120}-1)}{R-1}$

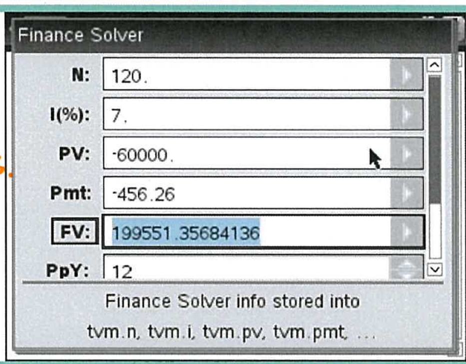
$= \$558974.01$ is new lump sum.

Alternatively, use the Finance Solver. We need to split it into the first 10 years and then the last 10 years

Using the Financial Solver, Enter the following:

n (N:) = 120
r (I(%)): = 7
P (PV:) = −60000
Pmt: = −456.26   use many decimals
FV: = ______
PpY: = 12
CpY: = 12

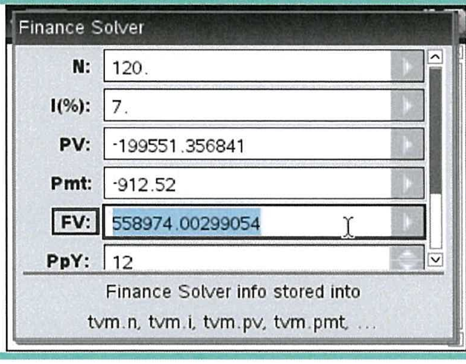
Place the cursor on FV: Press ENTER to solve.



Using the Financial Solver, Enter the following:

n (N:) = 12
r (I(%)): = 7
P (PV:) = −199551.35...
Pmt: = −912.52
FV: = ______
PpY: = 12
CpY: = 12

Place the cursor on FV: Press ENTER to solve.



Write Answer: The new lump sum is \$558974.00

c) How much extra could Andrew expect if the interest rate from part **b** is increased to 9% p.a. (from the final 10 years) compounded monthly? Round the answer correct to the nearest \$1000.

Now $r = 9$ and $R = 1 + \frac{9/12}{100} = 1.0075$

so $V_{120} = \$199551.36 \times (1.0075)^{120} + \frac{912.52\left((1.0075)^{120} - 1\right)}{1.0075 - 1}$

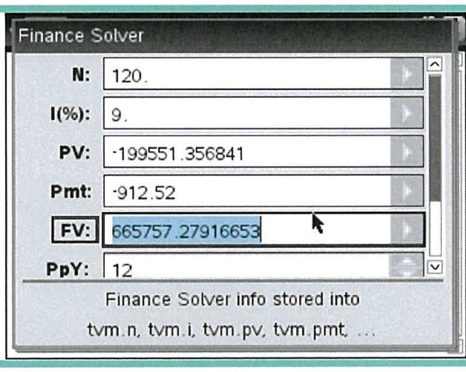
$= \$665757.29$ is the new lump sum.

Alternatively, use the Finance Solver

Using the Financial Solver, Enter the following:

n (N:) = 120
r (I(%)): = 9
P (PV:) = −199551.36
Pmt: = −912.52
FV: = ______
PpY: = 12
CpY: = 12

Place the cursor on FV: Press ENTER to solve.



Write Answer: If the interest rate is increased to 9% for the final 10 years, Andrew would expect an extra \$107,000 (nearest \$1000).
(665757.28 − 558974 = 106783.28)

Retirement

Once a person retires, they can receive a lump sum from their Superannuation fund, this lump sum can be transferred or rolled over to a suitable annuity. This annuity will provide a regular income to live on.

There are two options:

1. Perpetuities – an annuity that provides regular payments forever. The benefits of this is it will provide income to the retiree no matter how long they live and it can be willed to relatives, who will collect the same annuity indefinitely.
2. Annuity – reducing balance. The fund manager borrows the money and pays the retiree a regular income for a specified period of time. The disadvantage is if the retiree out lives the term of the reducing balance annuity, the money will run out.

Worked Example 27

Jarrod is aged 50 and is planning to retire at 55. His annual salary is $70 000 and his employer contributions are 9% of his gross monthly income. Jarrod also contributes a further $500 a month as a salary sacrifice (that is, he pays $500 from his salary to the superannuation fund. The superfund has been returning an interest rate of 7.2% p.a. compounded monthly and his current balance in the superfund is $255 000.

a) Calculate Jarrod's total monthly contributions to the superannuation fund.

Total Monthly is the $500 from Jarrod and 9% from Boss.

9% of $70,000 = $\frac{9}{100} \times 70{,}000 = \$6300$ which is $\frac{\$6300}{12} = \$525$ monthly.

So, total monthly contribution is $500 + $525.

which is $1025 per month.

b) Calculate the lump sum that he can receive for his planned retirement at age 55.

$PV = \$255000$

$d = \$1025$

$n = 5 \times 12 = 60$

$R = 1 + \left(\frac{7.2}{12} / 100\right) = 1.006$

$$V_{60} = 255{,}000 \times (1.006)^{60} + \frac{1025\left((1.006)^{60} - 1\right)}{1.006 - 1}$$

$$V_{60} = \$438{,}869.90$$

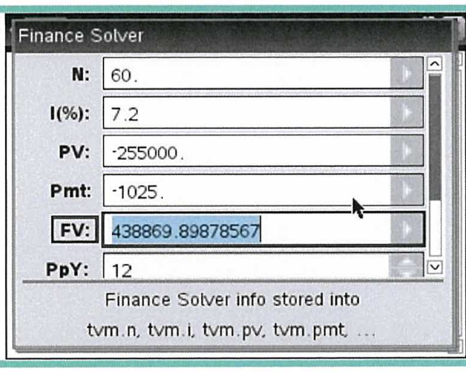
The lump sum available at 55 will be $438869.90.

Alternatively, use the Finance Solver

Using the Financial Solver, Enter the following:

- n (N:) = 60
- r (I(%)): = 7.2
- P (PV:) = -255000
- Pmt: = -1025
- FV: = ______
- PpY: = 12
- CpY: = 12

Place the cursor on FV: Press ENTER to solve.



Jarrod has two options for setting up an annuity to provide a regular income after he retires at 55.

1. A perpetuity that offers monthly payments at 8% p.a. compounded monthly.
2. A reducing balance annuity, also paid monthly at 8% p.a., compounded monthly.

c) Calculate the monthly annuity using options 1. Express the annual salary from this option as a percentage of his current salary.

Perpetuity Formula $d = \frac{V_0 r}{100}$, where $V_0 = \$438869.90$

$r = 8\%$.

$$\Rightarrow d = \frac{438869.90 \times 8}{100}$$

$$= \$35,109.59 \text{ per year.}$$

$$\Rightarrow \frac{35109.59}{12} = \$2925.80 \text{ per month.}$$

Percentage of current salary $\frac{\$35,109.59}{\$70000} \times 100\% = 50.16\%$

d) Calculate the monthly annuity using option 2 if the fund needs to last for 25 years. Express the annual salary from this option as a percentage of his current salary.

Annuity Formula $V_n = V_0 R^n - \frac{d(R^n - 1)}{R - 1}$.

State: $V_0 = 438869.90$, $d = ?$, $V_n = 0$, $R = 1 + \frac{8/12}{100} = 1.006$

and $n = 25 \times 12 = 300$

$\boxed{\text{CAS}}$ $\text{solve}\left(0 = 438869.90 \times (1.006)^{300} - \frac{d((1.006)^{300} - 1)}{(1.006 - 1)}, d\right)$

gives $d = \$3387.27$ monthly payment.

Alternatively, use the Finance Solver

Using the Financial Solver, Enter the following:

n (N:) = ________

r (I(%): = ________

P (PV:) = ________

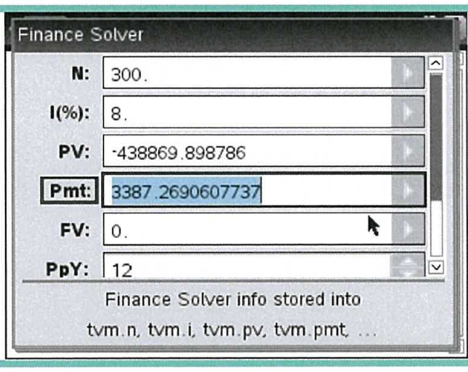
Pmt: = ________

FV: = ________

PpY: = ________

CpY: = ________

Place the cursor on Pmt: Press ENTER to solve.



For both we still need Percentage of current Percentage. calculation

$\$3387.27 \times 12 = \$40,647.24$ yearly.

Percentage $\frac{\$40647.24}{\$70,000} \times 100\% = 58.07\%$

The perpetuity will provide \$2925.80/month or 50.16% of his current salary. Whereas, the reducing balance annuity gives \$3387.27/month or 58.07% of current salary.

2010 FURMATH EXAM 2

Question 4

A home buyer takes out a reducing balance loan of $250 000 to purchase an apartment.
Interest on the loan will be calculated and paid monthly at the rate of 6.25% per annum.

a. The loan will be fully repaid in equal monthly instalments over 20 years.

i. Find the monthly repayment, in dollars, correct to the nearest cent.

ii. Calculate the total interest that will be paid over the 20 year term of the loan.
Write your answer correct to the nearest dollar.

1 + 2 = 3 marks

b. After 60 monthly repayments have been made, what will be the outstanding principal on the loan?
Write your answer correct to the nearest dollar.

1 mark

By making a lump sum payment after nine years, the home buyer is able to reduce the principal on his loan to $100 000. At this time, his monthly repayment changes to $1250. The interest rate remains at 6.25% per annum, compounding monthly.

c. With these changes, how many months, in total, will it take the home buyer to fully repay the $250 000 loan?

1 mark

27

2011 FURMATH EXAM 2

Question 4

Tania takes out a reducing balance loan of $265 000 to pay for her house.
Her monthly repayments will be $1980.
Interest on the loan will be calculated and paid monthly at the rate of 7.62% per annum.

a. **i.** How many monthly repayments are required to repay the loan?
Write your answer to the nearest month.

ii. Determine the amount that is paid off the principal of this loan in the first year.
Write your answer to the nearest cent.

1 + 1 = 2 marks

Immediately after Tania made her twelfth payment, the interest rate on her loan increased to 8.2% per annum, compounding monthly.
Tania decided to increase her monthly repayment so that the loan would be repaid in a further nineteen years.

b. Determine the new monthly repayment.
Write your answer to the nearest cent.

1 mark