7.7 Perpetuities

There a variety of ways to invest money, one is a managed fund, whereby, you invest an initial principal and hope the fund managers are able to invest the money to gain a positive return. In some years money may be lost and the percentage returns can vary from year to year.

Another option is to regularly contribute to the fund after the initial investment, thus increasing the principal and the interest earnt. This is an example of a first-order recurrence relation.

Worked Example 20

Jonathan invested $5000 in a managed fund that will earn an average of 8% p.a. over a 2 year period with interested calculated monthly. If Johnathan contributes $100 at the start of the second, third, fourth and fifth months, complete the table to find the value of his investment at the end of the fifth month.

$r = 8\%$ p.a. $= \frac{0.08}{12} = 0.0067$ per month.

so Interest on \$5000, $I = 5000 \times (0.0067)$
$= 33.33$

Time period	Principal ($)	Interest earned ($)	Balance ($)
1	5000	$I = 5000 \times \frac{0.08}{12} = 33.33$	$5000 + 33.33 = 5033.33$
2	5133.33	$I = 5133.33 \times \frac{0.08}{12} = 34.22$	$5133.33 + 34.22 = 5167.55$
3	5267.55	$I = 5267.55 \times \frac{0.08}{12} = 35.12$	$5267.55 + 35.12 = 5302.67$
4	5402.67	$I = 5402.67 \times \frac{0.08}{12} = 3602$	$5402.67 + 36.02 = 5438.69$
5	5538.69	$I = 5538.69 \times \frac{0.08}{12} = 36.92$	$5538.69 + 36.92 = 5575.61$

A perpetuity is an annuity where a permanently invested sum of money provides regular repayments that continue forever. Scholarships or grants offered to students at University are provided by funds known as perpetuities. Wealthy people who wish to support a worthy cause set up perpetuities.

The funds last for an indefinite period of time as the amount paid out is the same as the interest earned on the initial lump sum deposited. The balance of the amount invested does not change and is the same for an indefinite period. — compare this to a interest only loan.

$$d = \frac{V_0 r}{100}$$

where

d = the amount of the regular payment per period ($)

V_0 = the principal ($)

r = the interest rate per period (%).

Note: The period of the regular repayments must be the same as the period of the given interest rate. Finance Solver can be used. As the principal does not change, the PV (negative cash flow) and the FV (positive cash flow) are entered as the same amount but with opposite signs.

Worked Example 21 Multiple Choice

Robert wishes to use part of his wealth to set up a scholarship fund to help young students from his town further their education at university. Robert invests $200 000 in a bond that offers a long-term guaranteed interest rate of 4% p.a. If the interest is calculated once a year, then the annual amount provided as scholarship will be:

A $188 000 **B** $288 000 **C** $666.67 **D** $8000 **E** $4000

Write down the perpetuity formula $d = \frac{V_0 r}{100}$

$V_0 =$ _____

$r =$ _____ % p.a.

$d = \frac{\$200{,}000 \times 4}{100} = \$8{,}000$

The annual amount provided for the scholarship is $8,000. Therefore D is the correct answer.

Finding V_0 and r

*If the frequency of the payments each year is **not** the same as the compounding period of the given interest rate, then Finance Solver is to be used with different values for PpY and CpY.*

Note: The principal must be known to use Finance Solver. Finance Solver gives the interest rate per annum

The perpetuity formula can be transposed to:

$$V_0 = \frac{100 \times d}{r} \text{ and } r = \frac{100 \times d}{V_0}$$

Worked Example 22

A Rotary Club has $100 000 to set up a perpetuity as a grant for the local junior sporting clubs. The club invests in bonds that return 5.2% p.a. compounding annually.

a) Find the amount of the annual grant.

$V_0 = \$100\,000$

$r = 5.2\%$ p.a.

$d = \frac{V_0 r}{100} = \frac{\$100000 \times 5.2}{100} = \$5{,}200$

The amount of the annual grant is $5,200.

Alternatively, use the Finance Solver

Using the Financial Solver, Enter the following:

n (N:) = 1

r (I(%): = 5.2

P (PV:) = -100,000

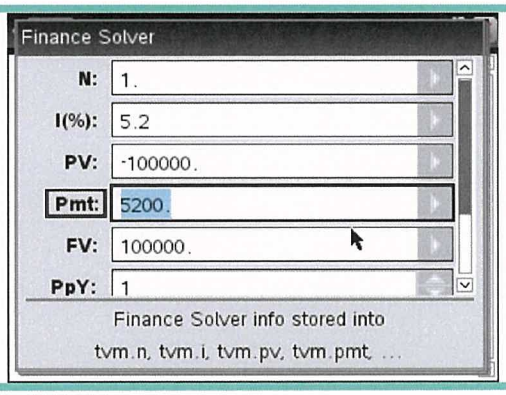
Pmt: = _____

FV: = 100,000

PpY: = 1

CpY: = 1

Place the cursor on Pmt: Press ENTER to solve.



b) What interest rate (compounding annually) would be required if the perpetuity is to provide $6000 each year?

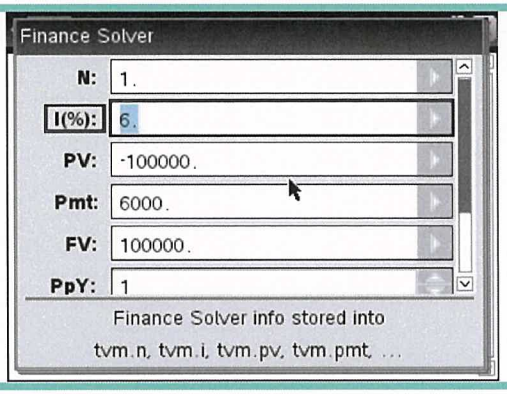
$V_0 = \$100\,000$
$d = \$6,000$
$r = \frac{100 \times d}{V_0} = \frac{100 \times 6,000}{100\,000} = 6\%$ p.a.

Alternatively, use the Finance Solver

Using the Financial Solver, Enter the following:

n (N:) = _____
r (I(%): = _____
P (PV:) = _____
Pmt: = _____
FV: = _____
PpY: = _____
CpY: = _____

Place the cursor on I(%): Press ENTER to solve.



For a $100,000 perpetuity to provide $6,000 a year, the bonds need to offer an interest rate of 6% p.a.

The Rotary Club wants to investigate other possible arrangements for the structure of the grant.

c) How much extra would the annual grant amount to if the original interest rate was compounded monthly?

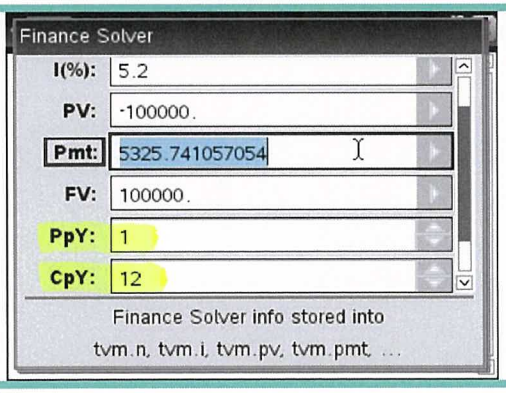
Using, the original values of $r = 5.2$, $PV = -100\,000$ and $n = 1$ and $FV = 100,000$ then pmt is:

Using the Financial Solver, Enter the following:

n (N:) = _____
r (I(%): = _____
P (PV:) = _____
Pmt: = _____
FV: = _____
PpY: = _____
CpY: = _____

Note: CpY and PpY differ.

Place the cursor on Pmt: Press ENTER to solve.



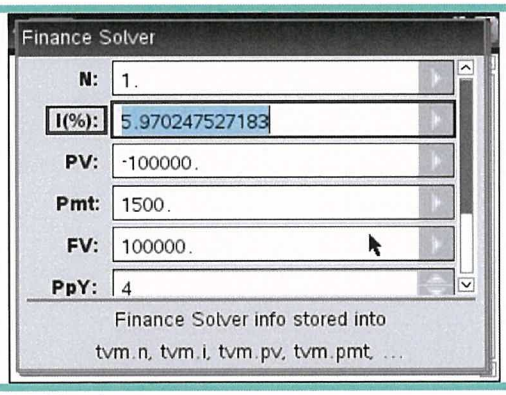
If the interest was compounded monthly, the annual growth would be $\$$ _____ $- \$5,200 =$ _____ extra.

d) What interest rate (compounded monthly) would be required to provide 4 equal payments of $1500 every 3 months? Give your answer correct to 2 decimal places.

Using the Financial Solver, Enter the following:

n (N:) = _____
r (I(%): = _____
P (PV:) = _____
Pmt: = _____
FV: = _____
PpY: = _____
CpY: = 12

Place the cursor on I(%):, Press ENTER to solve.



An interest rate of 5.97% p.a. compound monthly is needed to provide four payments of $1500

Worked Example 23

A benefactor of a college has been approached to provide a Year 7 scholarship of $1000 per term. He is able to get a financial institution to offer a long-term interest rate of 8% per annum. What is the principal that needs to be invested?

Perpetuity formula

$$V_0 = \frac{100 \times d}{r}$$

d = \$1000 per term (4 terms per year).

R = 8% p.a.

= 8/4 = 2% per term.

so $V_0 = \frac{100 \times \$1000}{2}$

= \$50,000

The principal that needs to be invested to provide a scholarship of \$1000 per term at an annual rate of 8% is \$50,000.