7.6 Effective annual interest rate

We have considered paying off a loan at a set interest rate, however, we have found the amount of interest paid would vary with different compounding terms (daily, weekly, monthly etc). The **effective annual interest rate** is used to compare the annual interest between loans with these compounding terms. different.

To calculate the effective annual interest rate, use the formula:

$$r = \left(1 + \frac{i}{n}\right)^n - 1$$

where

$r =$ the effective annual interest rate

$i =$ the nominal rate, as a decimal

$n =$ the number of compounding periods per year

For a loan of \$100 at 10% p.a. compounding quarterly over 2 years.
The effective annual interest rate:

$$\begin{aligned} r &= \left(1 + \frac{i}{n}\right)^n - 1 \\ &= \left(1 + \frac{0.10}{4}\right)^4 - 1 \\ &= 0.1038 \\ &= 10.38\% \end{aligned}$$

This means that the effective annual interest rate is actually 10.38% and not 10%.
The comparison between the two can be shown in the following table.

Period	Amount owing (\$)	Annual effective rate calculation (\$)
1	$100\left(1 + \frac{0.10}{4}\right) = 102.50$	
2	$102.50\left(1 + \frac{0.10}{4}\right) = 105.06$	
3	$105.06\left(1 + \frac{0.10}{4}\right) = 107.69$	
4 (Year 1)	$107.69\left(1 + \frac{0.10}{4}\right) = 110.38$	$100\left(1 + \frac{10.38}{100}\right)^1 = 110.38$
5	$110.38\left(1 + \frac{0.10}{4}\right) = 113.14$	
6	$113.14\left(1 + \frac{0.10}{4}\right) = 115.97$	
7	$115.97\left(1 + \frac{0.10}{4}\right) = 118.87$	
8 (Year 2)	$118.87\left(1 + \frac{0.10}{4}\right) = 121.84$	$100\left(1 + \frac{10.38}{100}\right)^2 = 121.84$

Worked Example 19

Jason decides to borrow money for a holiday. If a personal loan is taken over 4 years with equal quarterly repayments compounding at 12% p.a., calculate the effective annual rate of interest (correct to 2 decimal places).

Write down the values for i and n

$n = 4$ (quarterly)

$i = 0.12$ (12% as a decimal).

Now, $r_{effective} = \left(1 + \frac{i}{n}\right)^n - 1$

$$= \left(1 + \frac{0.12}{4}\right)^4 - 1$$

$$= 0.1255$$

$$= 12.55\%.$$

The effective annual interest rate is 12.55%, for a loan of 12%.