7.4 Reducing balance loans III

When paying off a loan it is often wise to follow its progress through the life of the loan. The **amortisation** of the loan can be tracked on a step-by-step basis by following the payments made, the interest and reduction in the principal. **Amortisation** is defined as the regular decrease in value (depreciation) of an asset or the paying off a debt over time through regular repayments.

Worked Example 10

Sharyn takes out a loan of $5500 to pay for solar heating for her pool. The loan is to be paid in full over 3 years with quarterly payments at 6% p.a.

a) Calculate the quarterly payment required.

quarterly for 3 years is 3×4

Calculate the value of n: n = 3years x 4 quarters = 12

Using the Financial Solver

Enter the following:

n (N:) = ________

r (I(%): = ________

P (PV:) = ________

Pmt: = ________

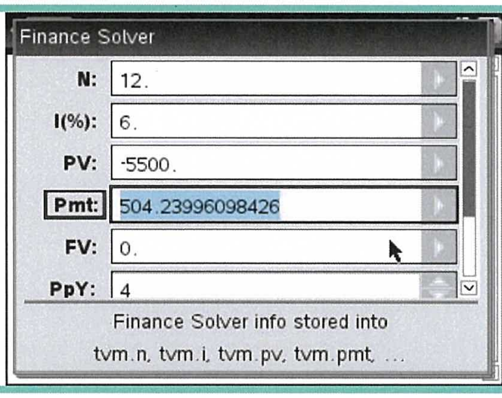
FV: = ________

PpY: = ________

CpY: = ________

Place the cursor on Pmt:

Press ENTER to solve.



In words...

The quarterly payments are $504.24

interest per quarter $= \frac{6}{100} \div 4 = 0.015$

Interest $= 5500 \times 0.015$ $= \$82.50$

place the payment amount in the payment column.

b) Complete an amortisation table for the loan with the following headings.

Payment	Principal outstanding ($)	Interest due ($)	Payment ($)	Loan outstanding ($)
1	5500	$ 82.50	504.24	5500 + 82.50 − 504.24 = 5078.26
2	$ 5078.26	$ 76.17	"	5154.45 − 504.24 = 4650.19
3	$ 4650.19	$ 69.75	"	4215.70
4	$ 4215.70	$ 63.24	"	3774.70
5	$ 3774.70	$ 56.62	"	3327.08
6	$ 3327.08	$ 49.91	"	2872.75
7	$ 2872.75	$ 43.09	"	2411.60
8	$ 2411.60	$ 36.17	"	1943.53
9	$ 1943.53	$ 29.15	"	1468.44
10	$ 1468.44	$ 22.03	"	986.23
11	$ 986.23	$ 14.79	"	496.78
12	$ 496.78	$ 7.45	"	− 0.01

Frequency of repayments

In this section the effect on the actual term of the loan, and on the total amount of interest charged, of making more frequent repayments. The value of the repayments will change, the actual outlay will not e.g. a $3000 quarterly repayment will be compared to a $1000 monthly repayments.

Worked Example 11

Tessa wants to buy a dress shop. She borrows $15 000 at 8.5% p.a. (debited prior to each repayment) of the reducing balance. She can afford quarterly repayments of $928.45 and this will pay the loan in full in exactly 5 years. One-third of the quarterly repayments gives the equivalent monthly repayment of $309.48. the equivalent fortnightly repayment is $142.84. Find:

a) monthly

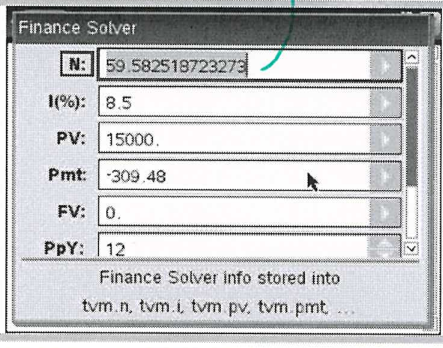
i) the term of the loan

The term of the loan is 5 years.

Using the Financial Solver
Enter the following:

n (N:) = _______
r (I(%): = _______
P (PV:) = _______
Pmt: = _______
FV: = _______
PpY: = _______
CpY: = _______

Place the cursor on N: Press ENTER to solve.



← 60 months
÷12
= 5 years.

ii) the amount still owing prior to the last payment if Tessa made repayments:

Using the Financial Solver: Enter the following:

n (N:) = _______
r (I(%): = _______
P (PV:) = _______
Pmt: = _______
FV: = _______
PpY: = _______
CpY: = _______

FV = $179.27360

so, the amount owing prior to the final payment is $179.27

b) fortnightly

i) the term of the loan

Using the Financial Solver: Enter the following:

n (N:) = ?
r (I(%): = _______
P (PV:) = _______
Pmt: = _______
FV: = _______
PpY: = _______
CpY: = _______

n = 129 fortnights.
which is 4 years and 25 fortnights.
(There are 26 fortnights per year.

ii) the amount still owing prior to the last payment if Tessa made repayments:

Using the Financial Solver: Enter the following:

n (N:) = _______
r (I(%): = _______
P (PV:) = _______
Pmt: = _______
FV: = ?
PpY: = _______
CpY: = _______

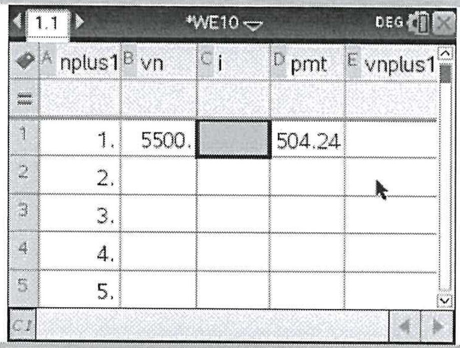
The amount still owing prior to the last payment is $120.64

Worked Example 10 on CAS calculator

- Enter the labels "n+1", "V_n", "I" (for interest), "Pmt" (for Payment), "V_{n+1}"

Note: You can't use + on the CAS so spell it out

- Next enter 1 to 12 in column A, and the starting values for $V_n=b1=5{,}500$, $Pmt=d1=504.24$ in cells *b1* and *d1* respectively.



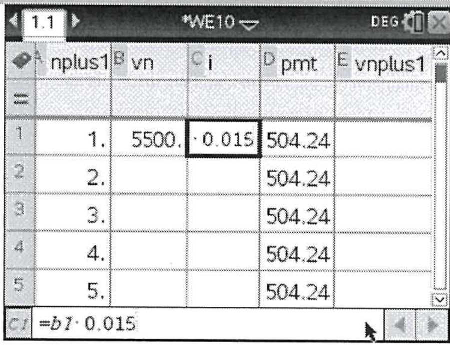
- In cell *c1* insert the equation

$$= b1\times 0.015$$

Note: *where 0.015 is r the interest rate per quarter*$\left(=\frac{6}{100}\div 4\right)$

- In cell d1

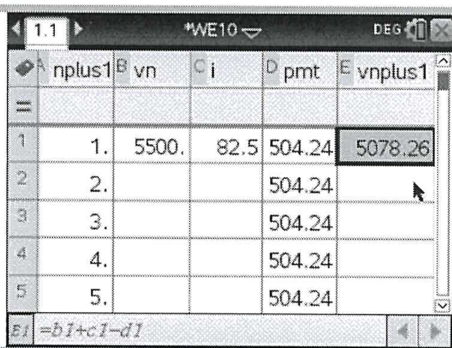
  Fill down (Menu, 3, 3)



- In *e1* enter the formula

$$= (b1 + c1) - d1$$

Which is (V_n + I)-pmt

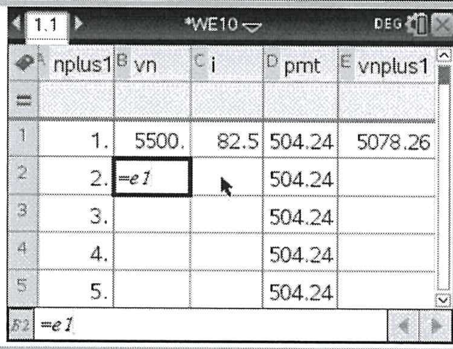


- In b1 enter

  $=e1$

  This is just using the previous answer as the starting value of the next.
- Now fill down the equations of cells *b2*, *c1* and *e1*, downward for each of columns b, c and e.

Note: the "-" you get is because the cell needs another cell filled to get the answer for this cell. **Don't Panic** it should work out.

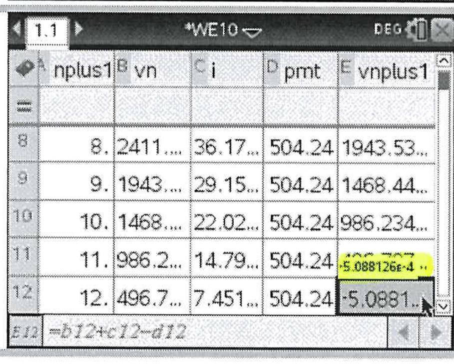


Last step is to copy the answers.

NOTE: The value of V_{12} is ***NOT*** -5.0881 ***BUT***

-0.0050881 or -0.01

Put your cursor on the box to make sure!!!!!!!!!!!



While the outlay will be the same, the term of the loan will be reduced when repayments are made more often. We can determine the savings for the loan. In such a case, the final (partial) repayment is considered separately as the interest charged is less in comparison to the rest of the repayments.

Total interest = total repayments − principal repaid.

Worked Example 12

In Worked example 11, Tessa's $15 000 loan at 8.5% p.a. gave the following three scenarios:

1. quarterly repayments of $928.45 for 5 years
2. monthly repayments of $309.48 for 59 months with $179.27 still outstanding
3. fortnightly repayments of $142.84 for 128 fortnights with $120.64 still owing.

Compare the total interest paid by Tessa if she repaid her loan:

a) quarterly
b) monthly
c) fortnightly

a) For quarterly repayments

Total Interest = Total repayments − principal repaid.
= $928.45 × (5×4) − 15,000
= $3569.00

b) For monthly repayments.

We need to find the interest on the $179.27 outstanding.

$r = \frac{8.5}{12} = 0.7083\%$ per month.

so interest on 179.27 is 0.7083 × 179.27 = $1.27

so Final payment = 179.27 + 1.27 = $180.54

Total Interest = 59 × 309.48 + 180.54 − 15000 = $3439.86

c) similar to b) but fortnightly. $r = \frac{8.5}{26} = 0.3269\%$

interest on $120.64 is $120.64 × 0.3269 = $0.39

Total Interest = ($142.84 × 128) + (120.64 + 0.39) − 15000
= $3404.55

Monthly Savings = $3569 − 3439.86 = $129.14 | Fortnightly savings = $3569.00 − 3404.55 = $164.45

Savings increase when the frequency of repayments increase, this occurs as the amount owed is reduced more frequently and so the amount of interest charged is slightly less.

Changing the rate

Over the term of a loan, the interest rate is likely to change. The Reserve Bank of Australia, the main monetary authority of the Federal Government, is the overall guiding influence on monetary factors in the Australian economy, it indirectly controls the interest rates, financial institutions charge.
Different financial institutions may have different interest rates and there is interest rate variation in an institution depending on the type of loan. Smaller loans, such as personal loans, have higher rates compared to home loans.
In this section, the effect of changing interest rates on the term of the loan and the total interest paid will be reviewed.

Worked Example 13

A reducing balance loan of $18 000 has been taken out over 5 years at 8% p.a. (adjusted monthly) with monthly repayments of $364.98.

a) What is the total interest paid?

Total Interest = total repayments − Principal repaid.
= $364.98 × 60 − $18,000
= $3898.80

The total interest paid is $3898.80.

b) If, instead, the rate was 9% p.a. (adjusted monthly) and the repayments remained the same, what would be:
i) the term of the loan

Using the Financial Solver
Enter the following:
n (N:) = ?
r (I(%): = 9
P (PV:) = 18,000
Pmt: = −364.98
FV: = 0
PpY: = 12
CpY: = 12

We wish to find the term of the loan or the "N"

N = 61.81 months = 62 months.

So, the term of the loan would now be 5 years, 2 months.

ii) the total amount of interest paid?

Using the Financial Solver
Enter the following:
n (N:) = 61
r (I(%): = 9
P (PV:) = 18000
Pmt: = −364.98
FV: = −293.886672
PpY: = 12
CpY: = 12

Now, because the term is over 61 repayments, BUT less than 62. We need to calculate the amount owing after 61 repayments.

FV (after 61 repayments) = $293.89

Interest per month = $\frac{9}{12}$ = 0.75%
Interest on $293.89 = 0.75 × 293.89.
= $2.20

⇒ Final repayment = 293.89 + 2.2 = $296.08

Total Interest = 364.98 × 61 + 296.08 − 18000 = $4559.86.

Worked Example 14

Natsuko and Hymie took out a loan for home renovations. The loan of $42 000 was due to run for 10 years and attract interest at 7% p.a., debited quarterly on the outstanding balance. Repayments of $1468.83 were made each quarter. After 4 years the rate changed to 8% p.a. (debited quarterly). The repayment value did not change.

a) Find the amount outstanding when the rate changed.

Using the Financial Solver
Enter the following:
n (N:) = 16
r (I(%): = 7
P (PV:) = 42000
Pmt: = 1468.83
FV: = ?
PpY: = 4
CpY: = 4

We need to find FV after 4 years
Since, the repayments are quarterly
the number of repayments is
n = 4 years × 4 quarters
= 16.

The amount outstanding when the rate changed is $28,584.36.

b) Find the actual term of the loan.

Using the Financial Solver
Enter the following:
n (N:) = ?
r (I(%): = 8
P (PV:) = $28,584.36
Pmt: = -1468.83
FV: = 0
PpY: = 4
CpY: = 4

So, now we need to pay $ 28,584.36 at a rate of 8 %.

n = 24.896 or 25 quarters
which is $6\frac{1}{4}$ years

Total term of the loan is
4 years + $6\frac{1}{4}$ years = $10\frac{1}{4}$ years.

c) compare the total interest paid to what it would have been if the rate had remained at 7% p.a. for the 10 years.

For 10 years at 7% Total Interest = $1468.83×40 - 42000
= $16753.20

Now, for the 8% period of 24.896 quarters
Find FV after 24 repayments is F = -1291.67
rate per quarter is $\frac{8}{4} = 2\%$ 2% of 1291.67 = $25.83
So, final repayment is $1291.61 + 25.83 = $1317.44
Total Interest = $1468.83×16 + $1468.83×24 + 1317.44 - 42000
= $18070.64

Interest difference = $18070.64 - $16753.20 = $1317.44

Interest only loans

Interest only loans are loans where the borrower makes only the minimum repayment equal to the interest equal to the interest charged on the loan. As the principal and amount owing is the same for the period of this loan, we can use the simple interest formula or CAS. When using the Financial Solver the present value (PV) and future value (FV) are the same (with the FV negative to indicate it is owed to the bank.

Note: Future value is negative to indicate the money is owed to the bank.

This type of loan is used by investors of shares and/or property or people experiencing financial difficulties and seek short-term relief.

Worked Example 15

Jade wishes to borrow $40 000 to invest in shares. She uses an interest only loan to minimise her repayment and hopes to raise a capital gain when she sells the shares at a higher value. The term of the loan is 6.9% p.a. compounded monthly with monthly repayments equal to the interest charged.

a) calculate the monthly interest-only repayment.

PV = $40,000
r = 6.9 %
FV = -$40,000
PpY = 12
CpY = 12

and N = 1 as all we need is the 1st payment.

The monthly repayment, for an interest only loan is $230.00

b) If, in 3 years, she sells the shares for $50 000, calculate the profit she would make on this investment strategy.

A Capital Gain is when the selling price is greater than all the expenses of the loan. (ie interest, repayments etc).

So, Capital Gain = selling price - purchase price.
= $50,000 - $40,000
= $10,000

Now Total Interest = repayment × number of repayments
= $230 × (3×12)
= $8,280

So Profit = Capital gain - Loan cost
= $10,000 - $8,280
= $1720

Jade will make a profit of $1720.