Point Cook Senior

Further Mathematics 2016
Core: RECURSION AND FINANCIAL MODELLING
Chapter 7 – Loans, investments and asset values

Key knowledge (Chapter 7)

- Amortisation of a reducing balance loan or annuity and amortisation tables
- Reducing balance loans, annuities, perpetuities and annuity investments.

Key skills

- Use a table to investigate and analyse on a step-by-step basis the amortisation of a reducing balance loan or an annuity, and interpret amortisation tables
- Using a CAS calculator, solve practical problems associated with compound interest investments and loans, reducing balance loans, annuities and perpetuities, and annuity investments.

Chapter Sections	Questions to be completed
7.2 Reducing balance loans I	1, 2, 4, 6, 8, 10, 12, 13, 15, 17
7.3 Reducing balance loans II	2, 4, 6, 8, 10, 12, 13, 18
7.4 Reducing balance loans III	2, 4, 6, 8, 10, 12, 14, 15, 20
7.5 Reducing balance and flat rate loan comparisons	2, 4, 5, 9, 10, 13, 17
7.6 Effective annual interest rate	2, 3, 4, 5, 8, 12,
7.7 Perpetuities	2, 4, 6, 8, 11, 13, 17
7.8 Annuity investments	2, 4, 6, 8, 10, 11, 14, 16, 18

More resources available at

http://pcsfurthermaths.weebly.com

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7.2 Reducing balance loans I

Introduction to Annuities

When we invest money with a financial institution such as a bank or credit union, the institution pays us interest as it is using our money to lend to others. Conversely, when we borrow money, we are using the financial institutions money and thus we are charged interest.

Interest is usually charged monthly by financial institution and repayments are made regularly by borrowers. The repayments are usually more than interest charged and therefore the amount owing reduces. Since the amount owing reduces the amount of interest charged reduces also.

The terms below are often used when talking about reducing balance loans:

- Principal, V_0 = amount borrowed ($)
- Balance, V_n = amount still owing ($)
- Term = life of the loan = (years)
- To discharge a loan = to pay off a loan (when V_n = $0)

'Interest only' loans exist where the repayments equal the interest added but the balance does not decrease. This option is available to people who wish to make the smallest payment possible such as property investors.

Annuities

An annuity is an investment that has regular and constant payments over a period of time e.g. Superannuation payments. Below is the recurrence relation that calculates the value of an annuity after each time period.

Note: This is the recurrence relation which gives the "next" value dependent on the "previous" value.

$$V_{n+1} = V_n R - d$$

where:

V_{n+1} = Amount left after $n + 1$ payments

V_n = Amount at time n

$R = \left(1 + \frac{r}{100}\right)$, where r is the interest rate per period

d = Payment amount

An annuity is over a fixed period of time. For example, a person retiring at 65, with $500,000 needs the money to last as long as they expect to live i.e. 20 years.

Worked Example 1

A loan of \$100 000 is taken out over 15 years at a rate of 7.5% p.a. (interest debited monthly) and is to be paid back monthly with \$927 instalments. Complete the table below for the first five payments.

$d = \$927$ $\qquad r = \frac{7.5}{12} = 0.625$

$V_0 = \$100000$ $\qquad R = \left(1 + \frac{0.625}{100}\right) = 1.00625.$

$n+1$	n	V_n	d	V_{n+1}
1	0	\$100 000 $= V_0$	\$927	$V_1 = 100000\left(1+\frac{0.625}{100}\right) - 927 = 99698$
2	1	\$99698 $= V_1$	\$927	$V_2 = 99698(1.00625) - 927 = 99394.1125$
3	2	\$99394.11 $= V_2$	\$927	$V_3 = 99394(1.00625) - 927 = 99088.32$
4	3	\$99088.32 $= V_3$	\$927	$V_4 = 99088(1.00625) - 927 = 98780.62$
5	4	\$98780.62 $= V_4$	\$927	$V_5 = 98780(1.00625) - 927 = 98471.00$

Worked Example 1 on CAS calculator

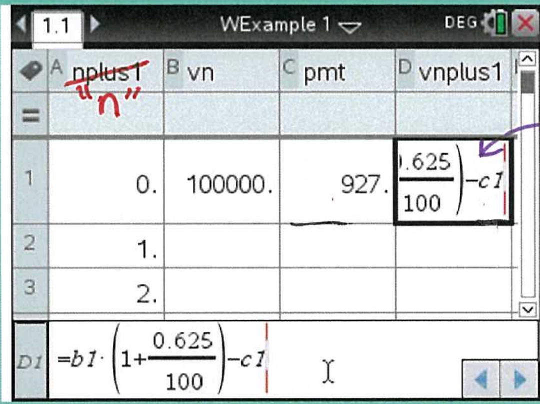
- Enter the labels "n+1", "V_n", "Pmt" (for Payment), "V_{n+1}"

Note: You can't use + on the CAS so spell it out

- Next enter 1 to 5 in column A, and the starting values for V_n=b1=100,000, Pmt=c1=927 in cells b1 and c1 respectively.
- In cell d1 insert the equation

$$= b1\left(1 + \frac{0.625}{100}\right) - c1$$

Note: where 0.625 is r the interest rate per period (7.5/12)

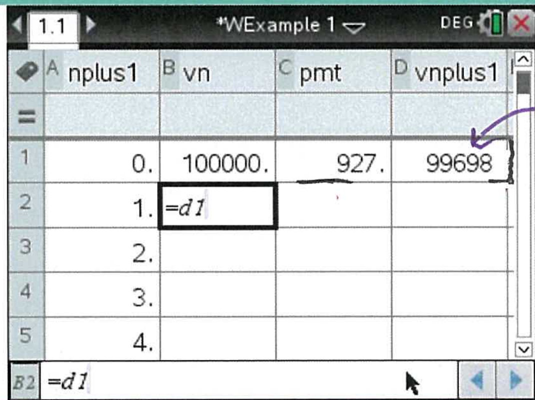


V_1

- In cell b2 enter

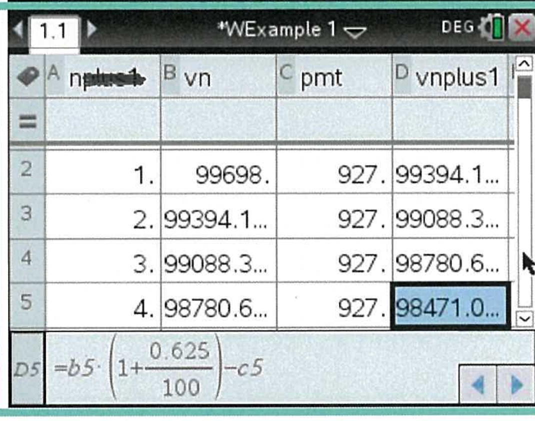
= d1

This is just using the previous answer as the starting value of the next.



V_1

Now fill down the equations of cells b2, c2 and d2, downward for each of columns b, c and d.



V_1 V_2 V_3 V_4

The Annuities formula

The annuities formula can be used to determine the amount of money still owing at any point of time during the term of a reducing balance loan. When someone borrows money from a financial institution that person is contracted to make regular payments (annuities) in order to repay the amount borrow in the agreed time period.

Note: This is for finding the value at anytime after "n" payments.

$$V_n = V_0R^n - \frac{d(R^n - 1)}{R - 1}$$

where:

V_0 = the amount borrowed (principal)

R = the compounding or growth factor for the amount borrowed

$= 1 + \frac{r}{100}$ (r = the interest rate per repayment period)

d = the amount of the regular payments made per period

n = the number of payments

V_n = the amount owing after n payments

Worked Example 2

A loan of $50 000 is taken out over 20 years at a rate of 6% p.a. (interest debited monthly) and is to be repaid with monthly instalments of $358.22. Find the amount still owing after 10 years.

$V_0 = \$50,000$, $d = \$358.22$, $n = 12 \times 10 = 120$

and $r = \frac{6}{12} = 0.5$, $R = 1 + \frac{r}{100} = 1 + \frac{0.5}{100} = 1.005$

$$V_{120} = V_0R^n - \frac{d(R^n - 1)}{R - 1} = 50,000 \times (1.005)^{120} - \frac{358.22(1.005^{120} - 1)}{1.005 - 1}$$

$$V_{120} = \$32264.98$$

* The amount still owed after 10 years is $32264.98

Worked Example 2 on CAS calculator

Use the solve function

Enter the equation

$$V_n = V_0R^n - \frac{d(R^n - 1)}{R - 1}$$

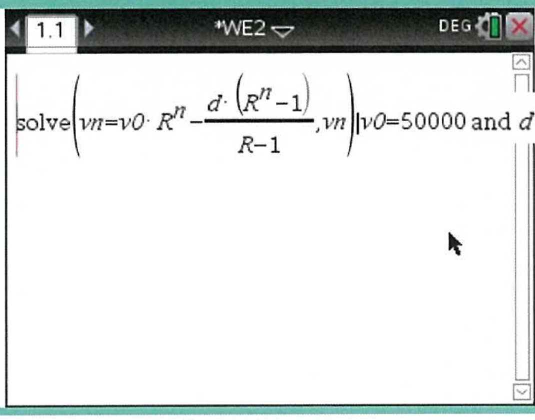
when:

V_0=50000

d=358.22

R=1.005

N=120



Top Tip: Save this equation and just change the values.

* Note: that even though 10 years is halfway through the 20 year loan, more than half is still owed.

The Financial Solver

Note: The Financial Solver on CAS can be used in annuities calculations in the same way it was used for compound interest calculations. – and is often easier!!!

Worked Example 3

Rob wants to borrow $2800 for a new sound system at 7.5% p.a., interest adjusted monthly.

a) What would be Rob's monthly repayment if the loan is fully repaid in 1 ½ years?

$V_0 = \$2800$, $n = 18$ months, $r = \frac{7.5}{12} = 0.625$.

$R = 1 + \frac{0.625}{100} = 1.00625$

Re-arranging the annuities formula for d is,

$$d = \frac{V_0 R^n (R-1)}{R^n - 1} = \frac{2800 \times 1.00625^{18}(1.00625-1)}{1.00625^{18} - 1} = \$164.95.$$

The monthly repayments are $164.95 over 18 months.

b) What would be the total interest charged?

Total Interest paid = repayments (d) × n − V_0

$= (164.95 \times 18) - 2800$

$= 2969.10 - 2800 = \$169.10$

The total interest on the $2800 loan over 18 months is $169.10

Worked Example 3 on CAS calculator

Using the Financial Solver

Enter the following:

n (N:) = 18

r (I(%):) = 7.5

V_0 (PV:) = 2800

Pmt: = ________

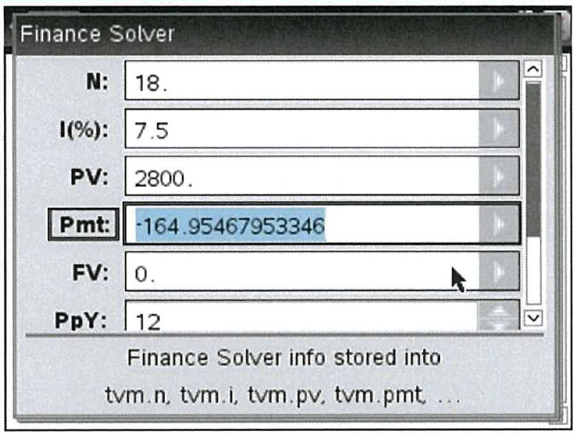
V_n (FV:) = 0

PpY: = 12

CpY: = 12

Place the cursor on Pmt:.

Press ENTER · to solve.



Total Interest paid = total repayments – amount borrowed

Total interest = 164.95 x 18 – 2800
= 2969.10 – 2800
= $169.10

Worked Example 4

Josh borrows $12 000 for some home office equipment. He agrees to repay the loan over 4 years with monthly instalments at 7.8% (adjusted monthly).

Find:

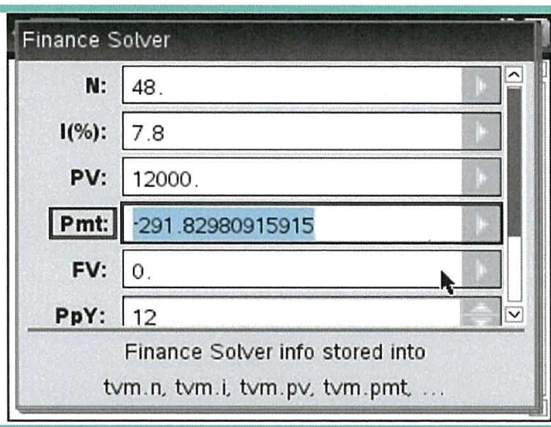
a) the instalment value

Calculate the value of n: n = 4 x 12 = 48

Using the Financial Solver
Enter the following:

- *n* (N:) = 48
- *r* (I(%:) = 7.8
- *P* (PV:) = 12000
- Pmt: = ______
- FV: = 0
- PpY: = 12
- CpY: = 12

Place the cursor on Pmt:
Press ENTER to solve.



The monthly repayment over the 4 year period is $291.83

b) the principal repaid and interest paid during the:

i) 10th repayment

To calculate this we need to find the difference between the 9th and 10th repayments. Using the CAS financial solver, this means we need to find the amount owed (FV) after the 9th and 10th payments.

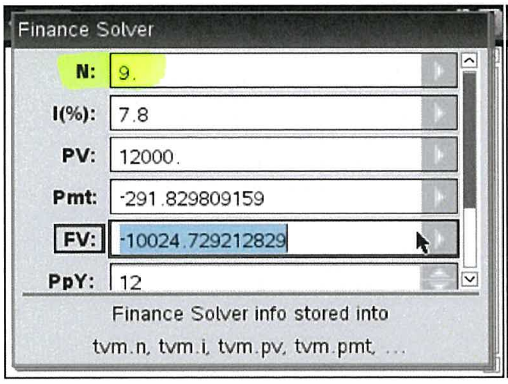
Using the Financial Solver
Enter the following:

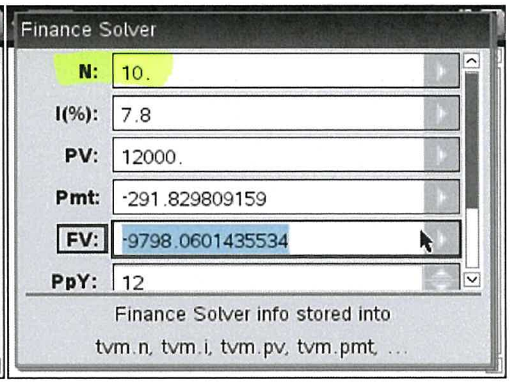
- *r* (I(%)) = 7.8
- *P* (PV): = 12 000
- Pmt: = -291.82
- (FV): = unknown
- PpY: = 12
- CpY: = 12

With

n (N:) = 9 & *n* (N:) = 10

Place the cursor on FV:
Press ENTER · to solve.





Principal owing after 9th repayment is $10024.73, Principal owing after 10th repayment is $9798.06.
So, the principal repaid during the 10th repayment is $10024.73 - $9798.06 = $226.67

If $291.83 is the monthly repayment and $226.67 is the principal repaid
then the interest paid is: $291.83 - $226.67 = $65.16

Answer in words, In the 10th repayment, $226.67 of the Principal is repaid and $65.16 interest is paid.

ii) 40th repayment

Using the Financial Solver

Enter the following:

r (I(%)): = 7·8

P (PV): = 12000

Pmt: = -291·83 from part (a)

(FV): = ________

PpY: = 12

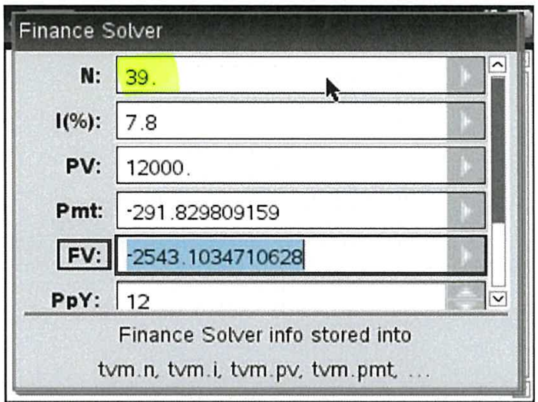
CpY: = 12

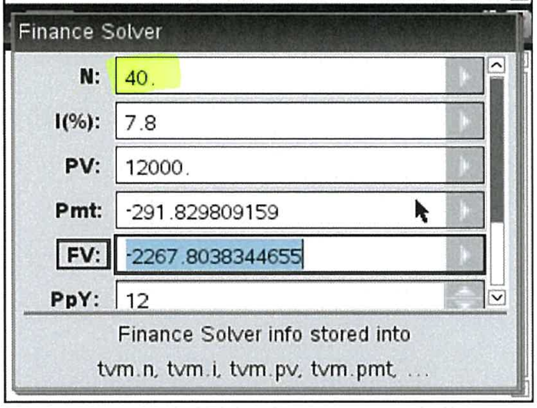
With

n (N:) = 39, 40

Place the cursor on FV:

Press ENTER to solve.





Principal owing after 39th repayment is \$2543.10, Principal owing after 40th repayment is \$2267.80.

So, the principal repaid during the 10th repayment is \$2543.10 - \$2267.80 = \$275.30

So, if \$291.83 is the monthly repayment and \$275.30 is the principal repaid then

\$291.83 - \$275.30 = \$16.53

In words, In the 40th repayment, \$275.30 of the Principal is repaid and \$16.53 interest is paid.

This makes sense!! At the 40th repayment there is less money owed so therefore there is less interest to pay.