Point Cook Senior

Further Mathematics 2016
Core: RECURSION AND FINANCIAL MODELLING
Chapter 6 – Interest and depreciation

Key knowledge

- the use of first-order linear recurrence relations to model flat rate and unit cost and reduce balance depreciation of an asset over time, including the rule for the future value of the asset after n depreciation periods
- the concepts of financial mathematics including simple and compound interest, nominal

Key skills

- demonstrate the use of a recurrence relation to determine the depreciating value of an asset or the future value of an investment or a loan after n time periods, including from first principles for $n \leq 5$
- use a rule for the future value of a compound interest investment or loan, or a depreciating asset, to solve practical problems

Chapter Sections	Questions to be completed
6.2 Simple interest	1, 2, 3, 4, 6, 7, 8, 9, 11a, 12ab, 13c, 14a, 16, 22
6.3 Compound interest tables	1, 2, 3, 4, 5, 6, 9, 11, 13
6.4 Compound interest formula	1, 3, 5, 6, 7ac, 8ac, 9ab, 10ac, 11, 12, 16ac
6.5 Finding rate or time for compound interest	1, 4, 5, 6, 7, 10, 11ab, 12ab, 13, 15
6.6 Flat rate depreciation	1, 4, 5, 7, 8, 11, 13, 15
6.7 Reducing balance depreciation	2, 3, 6, 7, 8, 10, 12, 14, 16, 18
6.8 Unit cost depreciation	1, 3, 5, 7, 9, 11, 13, 18

More resources available at

http://pcsfurthermaths.weebly.com

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6.2 Simple Interest

When you deposit money into a bank account, the bank is effectively borrowing money from you, which they then use. They pay you a small amount to "thank-you" for letting them use it. This is called interest. If the bank gives you interest of a *fixed amount* at *regular time periods*, this is called a **Simple Interest Investment**.

If you borrow money from the bank and you are charged a *fixed amount* of interest *at regular time periods* it is called a **Simple Interest Loan.**

Simple Interest is an example of linear growth. As we have seen in Chapter 5 linear growth can be expressed as first-order recurrence relation. The amount borrowed or invested is the starting value (V_0) or *Principal*. And with Simple Interest the rule is that a small amount is added at each step. This amount is usually a percentage of the Principal. This percentage is called the *interest rate*, and is expressed over a period of time. For example, 6% per annum (per year) or 0.05% per month etc.

Recurrence relation for Simple Interest

Let V_0 = Principal (the original amount invested)

Let r = be the percentage interest rate

Let V_n be the value of the loan or investment after n years

Simple interest can then be represented by a first-order linear recurrence relation.

$$V_{n+1} = V_n + d, d = \frac{V_0 \times r}{100},$$

where V_n represents the value of the investment after n time periods, d is the amount of interest earned per period, V_0 is the initial (or starting) amount and r is the interest rate.

So, the *Total* amount of a loan or investment is given by:

Total amount of loan or investment = initial amount or principal + interest

$$V_n = V_0 + I$$

where I is the Total Interest earned over the **entire time period**

$$I = \frac{V_0 rn}{100}$$

I = simple interest charged or earned ($)
V_0 = principal (money invested or loaned) ($)
r = rate of interest per period (% per period)
n = the number of periods (years, months, days) over which the agreement operates

Worked Example 1

$325 is invested in a simple interest account for 5 years at 3% p.a.

a) Set up a recurrence relation to find the value of the investment after n years.

$$d = \frac{V_0 \times r}{100} = \frac{325 \times 3}{100} = 9.75$$

First calculate the amount of Interest per period (d)

Use $V_0 = \$325$, $d = 9.75$ to write recurrence relation.

$V_0 = 325$, $V_{n+1} = V_n + 9.75$

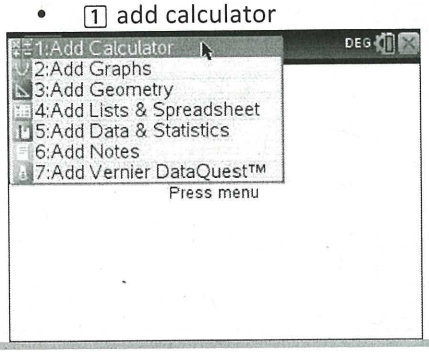
b) Use the recurrence relation from part (a) to find the value of the investment at the end of each of the first 5 years.

n	$n+1$	$V_n(\$)$	$V_{n+1}(\$)$
0	= 1	$V_0 = 325$	$V_{n+1} = 325 + 9.75 = 334.75 = V_1$
1	= 2	$V_1 = 334.75$	$V_2 = 334.75 + 9.75 = 344.50 = V_2$
2	3	$V_2 = 344.50$	$V_3 = 344.50 + 9.75 = 354.25 = V_3$
3	4	$V_3 = 354.25$	$V_4 = 354.25 + 9.75 = 364.00 = V_4$
4	5	$V_4 = 364.00$	$V = 364.00 + 9.75 = 373.75 = V_5$

Worked Example 1(b) on CAS calculator

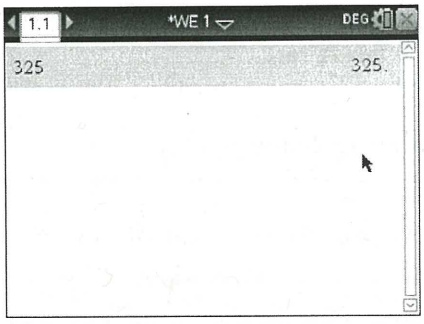
Start with a blank calculator page.
Press

- on Home
- 1 New document
- 1 add calculator

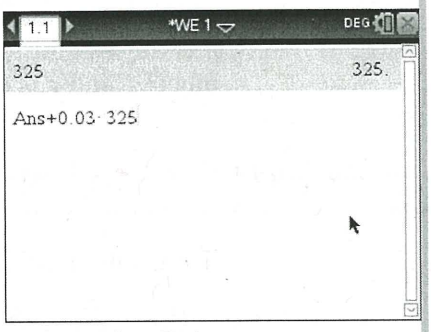


Enter the starting value

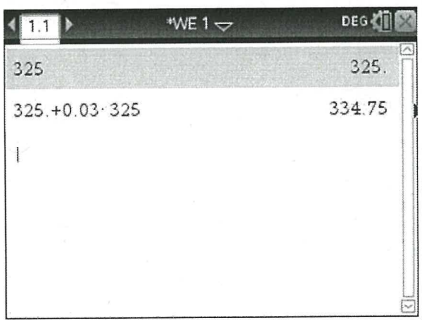
- Type 325 3 2 5
- press enter enter



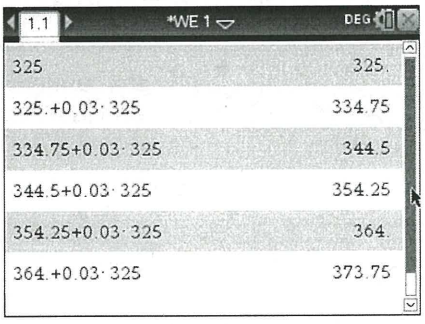
Next
type + . 0 3 × 3 2 5



Press enter enter
Note: when you press enter, the CAS converts ANS to the value of the previous answer (in this case 325)



Pressing enter repeatedly applies the rule "+0.03x325" to the last calculated value, in the process generating the amount of the investment at the end of each year as shown.



Worked Example 2

Jan invests $210 with building society in a fixed deposit account that paid 8% p.a. simple interest for 18 months.

a) How much did she receive after the 18 months?

n (number of periods) $= 1.5$ years*.

$V_0 = \$210$, rate $(r) = 8$

The interest per period (d)

$$d = \frac{V_0 \times r}{100} = \frac{210 \times 8}{100} = 16.8$$

Now to find the total interest $I = n \times d = 1.5 \times 16.8$

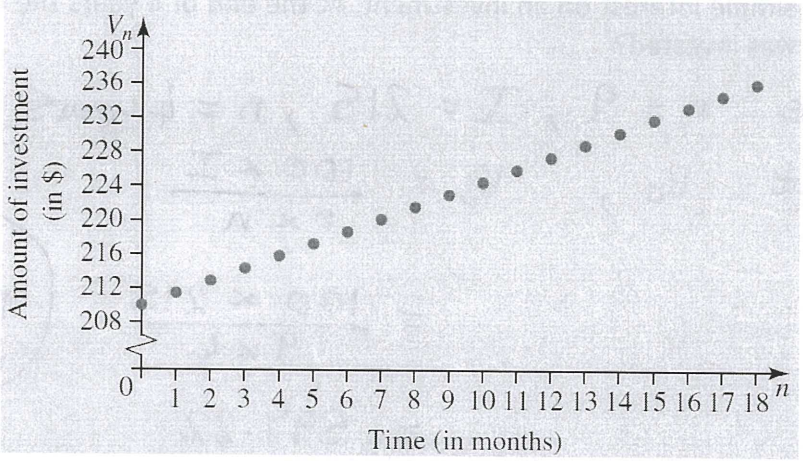
$= 25.20$

And the Total received is $V_n = V_0 + I$

$= 210 + 25.2 = \$235.20$

* same units (years) as the rate.

b) Represent the account balance for each of the 18 months graphically.



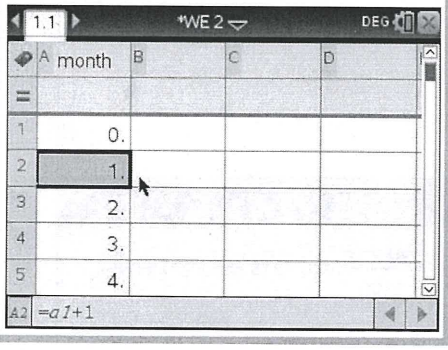
We need to work out the interest per month.

$$I_{month} = \frac{25.20}{18}$$

$= \$1.40$ per month.

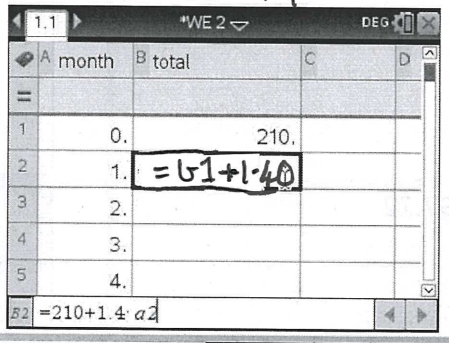
Worked Example 2(b) on CAS Calculator

Label column A "month"
Enter 0 in cell A1
In cell A2 enter: =a1+1
Fill down until the 18th month

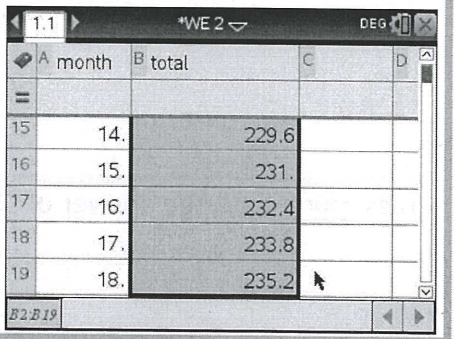


Label column B "total"
Enter $210 in cell b1
In the next cell (B2) enter the equation

$= b1 + 1.40$

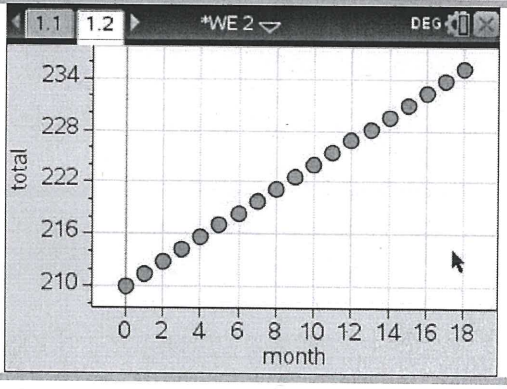


Now fill down this equation to the cells below.
Press
Menu [menu], data [3], fill [3]



Add a data and statistics page
[ctrl] [doc]

Put the "month" on the x axis and "total" on the y axis



Finding V_0, r and n

Transposed simple interest formula

To find the principal:	$V_0 = \frac{100 \times I}{r \times n}$
To find the interest rate:	$r = \frac{100 \times I}{V_0 \times n}$
To find the period of the loan or investment:	$n = \frac{100 \times I}{V_0 \times r}$

You could also use the solve function on the CAS.

Worked Example 3

A bank offers 9% p.a. simple interest on an investment. At the end of 4 years the total interest earned was \$215. How much was invested?

We have $r = 9$, $I = 215$, $n = 4$ years

We want V_0, $V_0 = \frac{100 \times I}{r \times n}$

$= \frac{100 \times 215}{9 \times 4}$

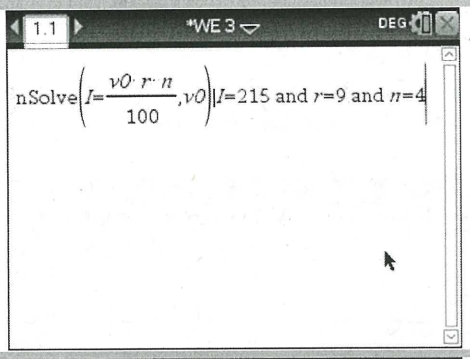
$= 597.22$

ANSWER.

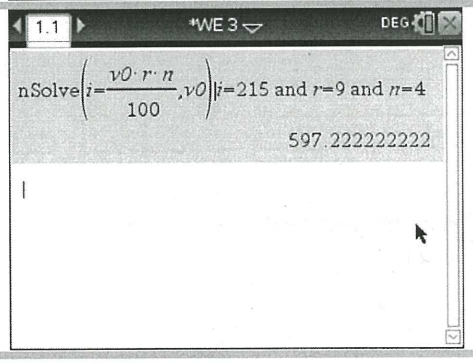
Amount invested is \$ 597.22.

Worked Example 3 on CAS calculator

On a calculator use the nSolve function, Enter the equation $I = \frac{V_0 \times r \times n}{100}$, and set the values of I, r and n using "|"



Press enter to get the answer of \$597.22



Worked Example 4

When $720 is invested for 36 months it earns $205.20 simple interest. Find the yearly interest rate.

We have $V_0 = \$720$, $n = 36$ months (3 years)

Interest total = $\$205.20$

Find the yearly interest rate. r

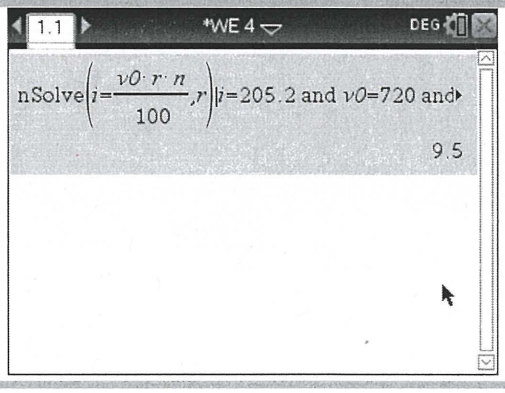
$$r = \frac{100 \times I}{V_0 \times n} = \frac{100 \times 205.20}{720 \times 3}$$

$$= 9.5$$

The yearly interest rate is 9.5%

Worked Example 4 on CAS calculator

On a calculator use the nSolve function, Enter the equation $I = \frac{V_0 \times r \times n}{100}$, and set the values of I, V_0 and n using "|", I=$205.20, V_0=720 and n=3



Worked Example 5

An amount of $255 was invested at 8.5% p.a. How long will it take, to the nearest year, to earn $86.70 in interest?

We have $V_0 = \$255$, $r = 8.5\%$ per year

$I = \$86.70$

We want n (to nearest year), $n = \frac{100 \times I}{V_0 \times r}$

$$= \frac{100 \times 86.70}{255 \times 8.5}$$

$$= 4$$

It will take 4 years

Worked Example 5 on CAS calculator

On a calculator use the nSolve function, Enter the equation $I = \frac{V_0 \times r \times n}{100}$, and set the values of I, V_0 and r using "|", I=$86.70, V_0=255 and r=8.5