VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2017

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER									Letter

FURTHER MATHEMATICS

Written examination 2

Monday 5 June 2017

Reading time: 2.00 pm to 2.15 pm (15 minutes)
Writing time: 2.15 pm to 3.45 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	7	7	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 33 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

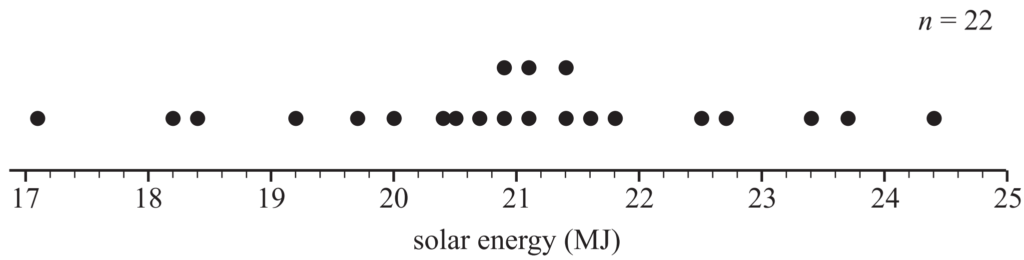
Data analysis

Question 1 (7 marks)

A 1 m^2 solar array is located at a weather station.

The total amount of energy generated by the solar array, in megajoules, is recorded each month.

The data for the month of February for the last 22 years is displayed in the dot plot below.

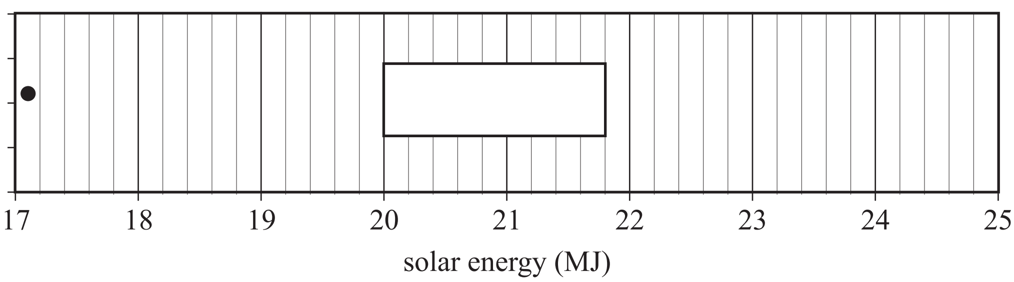


a. Determine the number of years in which the energy generated during February was greater than 23 MJ. 1 mark

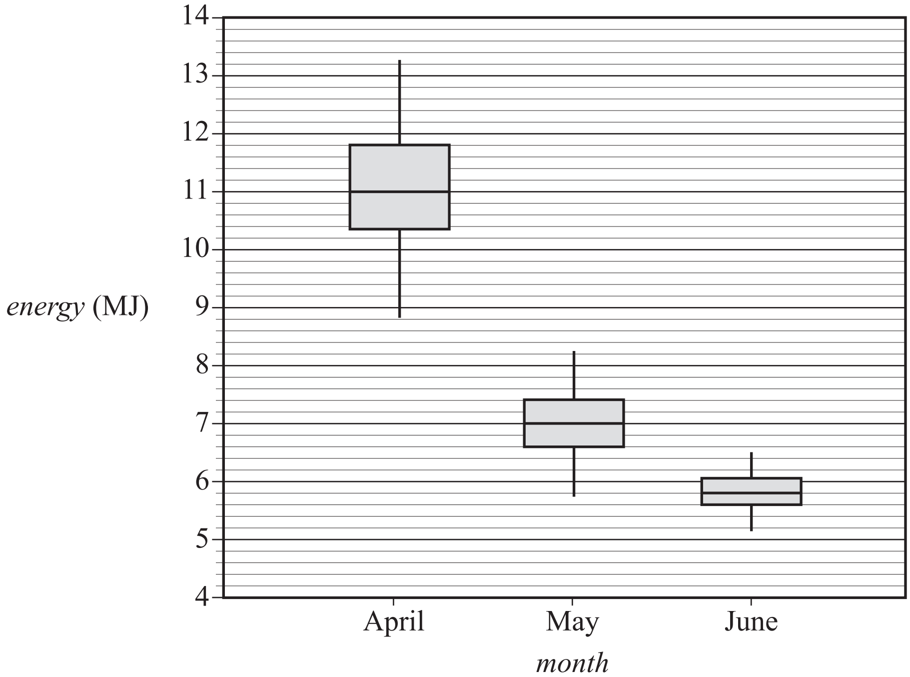
b. For the data in the dot plot above, the first quartile $Q_1 = 20$ and the third quartile $Q_3 = 21.8$

Show that the data value 17.1 is an outlier. 2 marks

c. Using the data in the dot plot, complete the boxplot below. 2 marks



d. The distribution of the amount of energy generated by the solar array for the months of April, May and June for the last 22 years is displayed in the parallel boxplots below.



The parallel boxplots suggest that the amount of energy generated is associated with the month of the year.

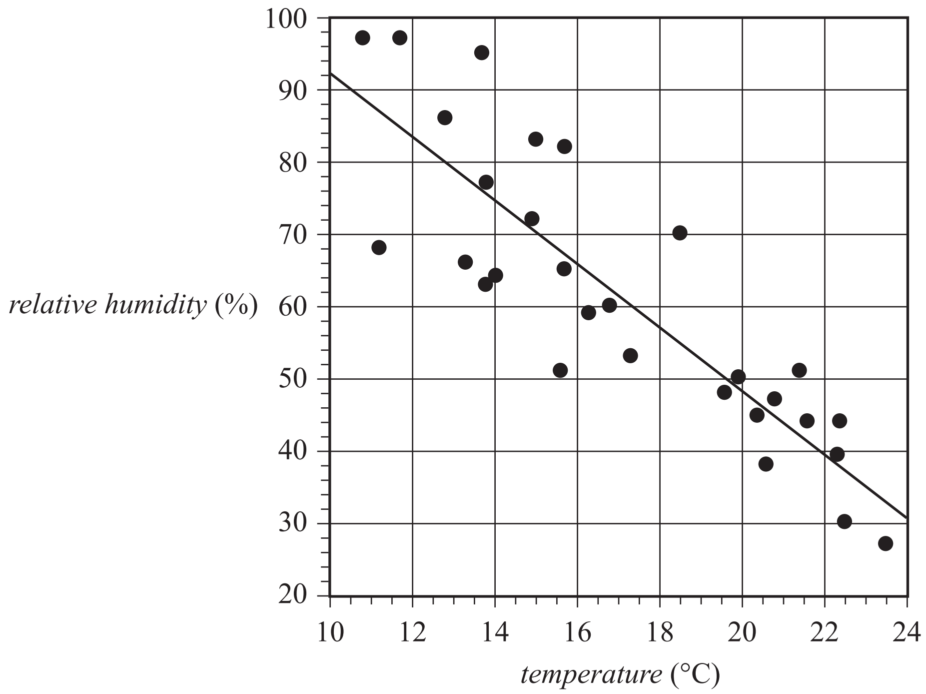
Explain why, quoting the values of an appropriate statistic. 2 marks

Question 2 (8 marks)

Two of the weather indicators collected at the weather station are temperature and relative humidity.

The scatterplot below shows *relative humidity* (%) plotted against *temperature* (°C) for the 29 days of February in a particular leap year. The measurements were taken at 3 pm each day.

A least squares line has been fitted to the scatterplot.



a. The coefficient of determination is 0.749

i. Write down the value of the correlation coefficient *r*.

Round your answer to two decimal places. 1 mark

ii. What percentage of the variation in *relative humidity* is **not** explained by the variation in *temperature*?

Round your answer to the nearest whole number. 1 mark

b. The equation of the least squares line is

relative humidity = 136 – 4.38 × *temperature*

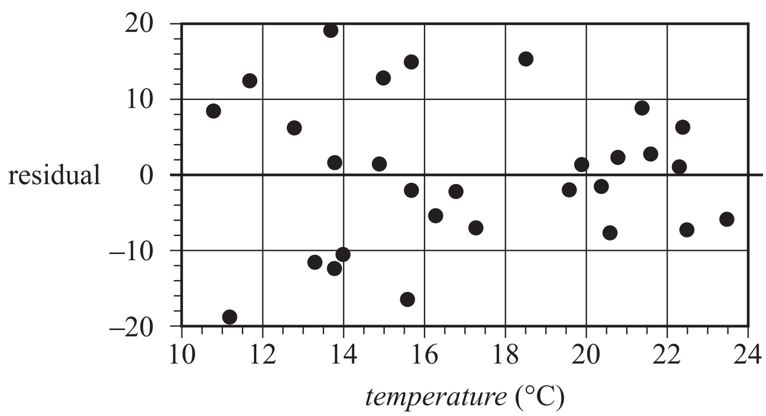
i. Write down the response variable. 1 mark

ii. Interpret the slope of the least squares line in terms of *relative humidity* and *temperature*. 2 marks

iii. When the *temperature* is 11.2 °C, the *relative humidity* is 68%.

Determine the residual value when the least squares line is used to predict the *relative humidity* at this *temperature*.
Round your answer to one decimal place. 2 marks

iv. The residual plot for the least squares line is shown below.

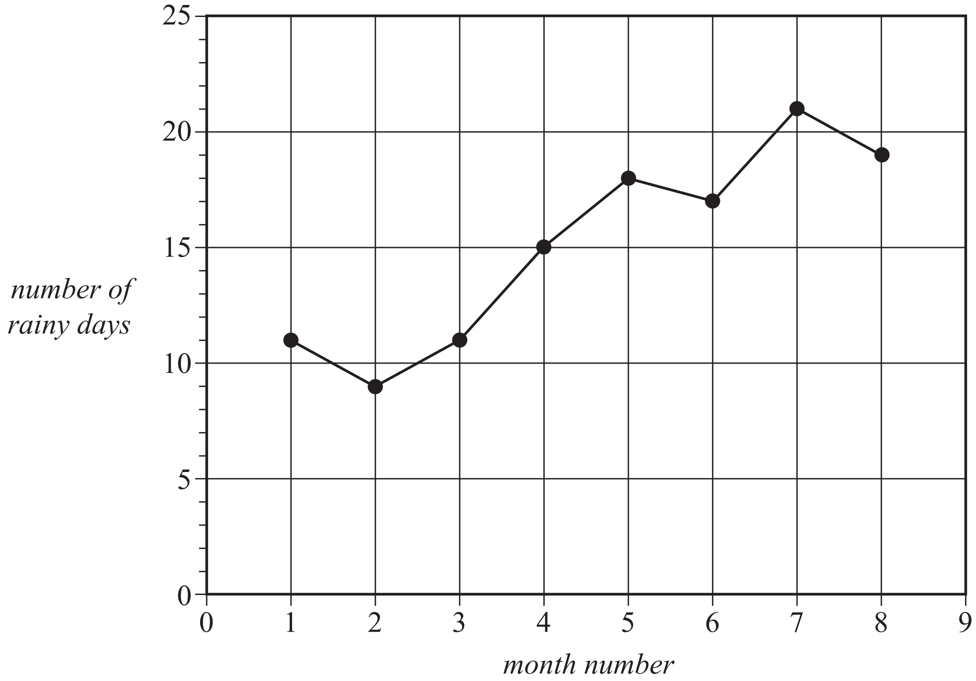


Does the residual plot support the assumption of linearity? Briefly explain your answer. 1 mark

Question 3 (5 marks)

The number of rainy days per month is also recorded at the weather station.

In the time series plot below, the *number of rainy days* per month is plotted for January (Month 1) to August (Month 8) in the same year.



a. Describe the trend in the time series plot. 1 mark

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The trend in the time series plot is to be modelled using a least squares line. The data used to construct this plot is given below.

Month number	1	2	3	4	5	6	7	8
Number of rainy days	11	9	11	15	18	17	21	19

b. Use the data above to determine the equation of the least squares line. Write the values of the intercept and slope in the boxes below. Round your answers to three significant figures. 3 marks

number of rainy days = [ ] + [ ] × *month number*

c. Draw the least squares line on the **time series plot above**. 1 mark

(*Answer on the time series plot above.*)

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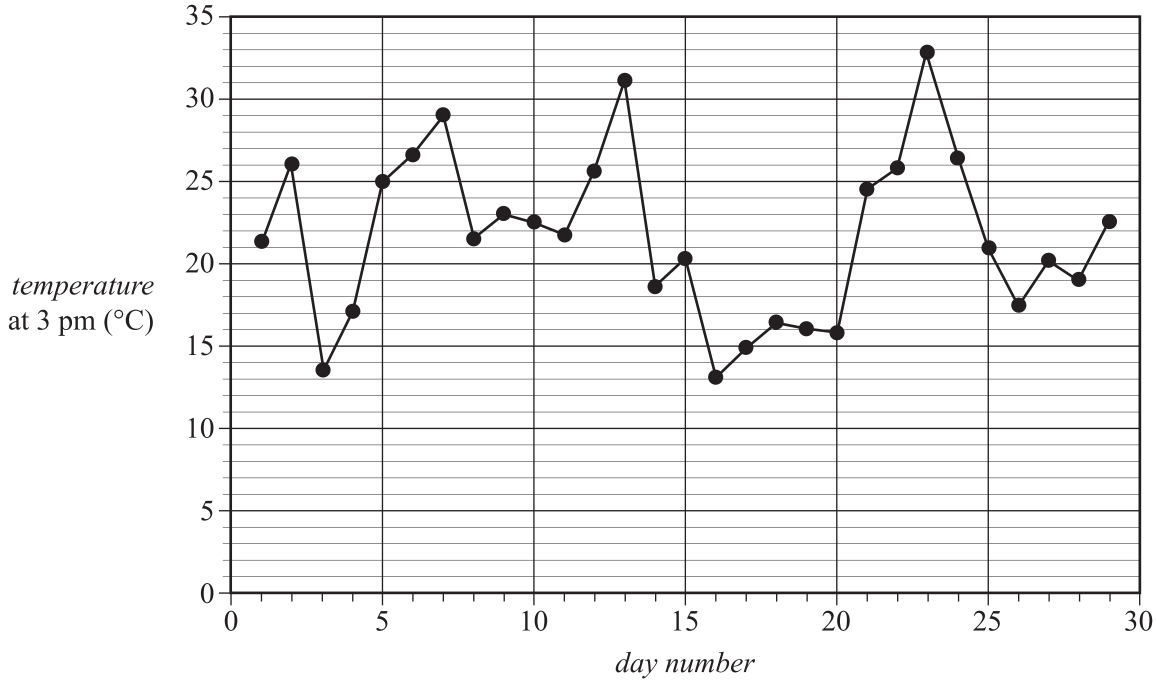
Question 4 (4 marks)

The time series plot below shows the *temperature* (°C) recorded at the weather station at 3 pm for the 29 days of February in a particular leap year.



a. Write down the range for the variable *temperature*.
Round your answer to the nearest whole number. 1 mark

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b. Determine the five-median smoothed *temperature* at 3 pm on day 14.
Round your answer to the nearest whole number. 1 mark

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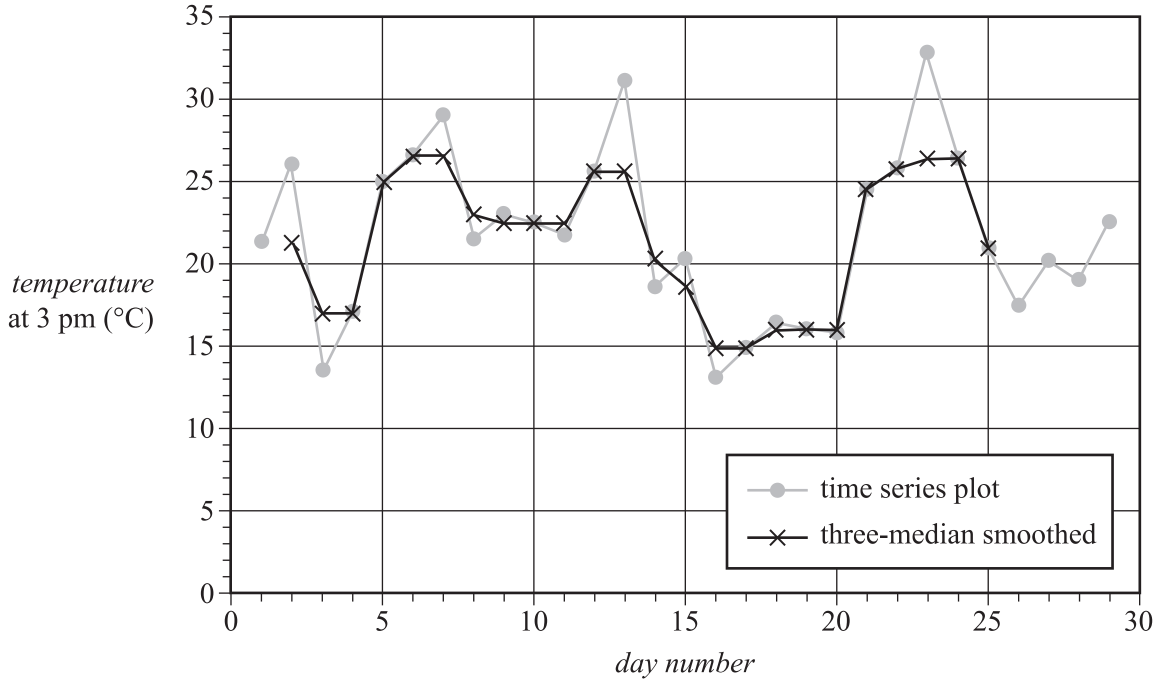
c. Three-median smoothing has now been used to smooth the time series plot up to day 25.

Complete the three-median smoothing by marking each remaining smoothed point with a cross (×) on the time series plot below. 2 marks



Recursion and financial modelling

Question 5 (5 marks)

The snooker table at a community centre was purchased for $3000.

After purchase, the value of the snooker table was depreciated using the flat rate method of depreciation.

The value of the snooker table, V_n, after *n* years, can be determined using the recurrence relation below.

$$V_0 = 3000, \qquad V_{n+1} = V_n - 180$$

a. What is the annual depreciation in the value of the snooker table? 1 mark

b. Use recursion to show that the value of the snooker table after two years, V_2, is $2640. 1 mark

c. After how many years will the value of the snooker table first fall below $2000? 1 mark

d. The value of the snooker table could also be depreciated using the reducing balance method of depreciation.

After one year, the value of the snooker table is $2760.

After two years, the value of the snooker table is $2539.20

i. Show that the annual rate of depreciation in the value of the snooker table is 8%. 1 mark

ii. Let S_n be the value of the snooker table after *n* years.

Write down a recurrence relation, in terms of S_{n+1} and S_n, that can be used to determine the value of the snooker table after *n* years using this reducing balance method. 1 mark

Question 6 (4 marks)

The community centre opened a savings account with Bank P.

Let P_n be the balance of the savings account n years after it was opened.

The value of P_n can be determined using the recurrence relation model below.

$$P_0 = A, \qquad P_{n+1} = 1.056 \times P_n$$

The balance of the savings account one year after it was opened was $1584.

a. Show that the value of A is $1500. 1 mark

b. Write down the balance of the savings account four years after it was opened. 1 mark

c. The balance of the savings account six years after it was opened was $2080.05

This $2080.05 was transferred into a savings account with Bank Q.

This savings account pays interest at the rate of 5.52% per annum, compounding monthly.

Let Q_n be the balance of this savings account n months after it was opened.

The value of Q_n can be determined from a rule.

Complete this rule by writing the missing values in the boxes provided below. 2 marks

$$Q_n = \boxed{} \times \boxed{}^{\,n}$$

Question 7 (3 marks)

The community centre has received a donation of \$5000. The donation is deposited into another savings account. This savings account pays interest compounding monthly.

Immediately after the interest has been added each month, the community centre deposits a further \$100 into the savings account.

After five years, the community centre would like to have a total of \$14 000 in the savings account.

a. What is the annual interest rate, compounding monthly, that is required to achieve this goal?

Write your answer correct to two decimal places. 1 mark

b. The interest rate for this savings account is actually 6.2% per annum, compounding monthly.

After 36 deposits, the community centre stopped making the additional monthly deposits of \$100.

How much money will be in the savings account five years after it was opened? 2 marks

END OF SECTION A

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents **Page**

Module 1 – Matrices

Question 1 (4 marks)

People pay to attend concerts at the Whiteoak Theatre.

They can choose their seats for each concert from three classes, *A*, *B* or *C*.

The table below shows the number of seats available in each class and the cost per seat.

Class	Number of seats available	Cost per seat ($)
A	100	45
B	340	35
C	160	30

a. The column matrix *N* contains the number of seats in each class.

$$N = \begin{bmatrix} 100 \\ 340 \\ 160 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

What is the order of matrix *N*? 1 mark

b. Matrix *W* contains the cost of each class of seat in the theatre.

$$W = \begin{matrix} A & B & C \\ [45 & 35 & 30] \end{matrix}$$

i. Determine the matrix product *WN*. 1 mark

ii. Explain what the matrix product *WN* represents. 1 mark

c. The number of seats that were sold for the first concert this year is shown in the table below.

Class	**Number of seats sold**	**Cost per seat ($)**
A	42	45
B	179	35
C	86	30

The information in the table is used to construct the matrix *P*, shown below.

$$P = \begin{bmatrix} 42 & 0 & 0 \\ 0 & 179 & 0 \\ 0 & 0 & 86 \end{bmatrix} \begin{bmatrix} 45 \\ 35 \\ 30 \end{bmatrix}$$

Matrix *P* contains the value of all seats in each class, in dollars, that were sold for the first concert this year.

A matrix product *MP* is found where $M = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$.

Explain what the matrix product *MP* represents. 1 mark

Question 2 (8 marks)

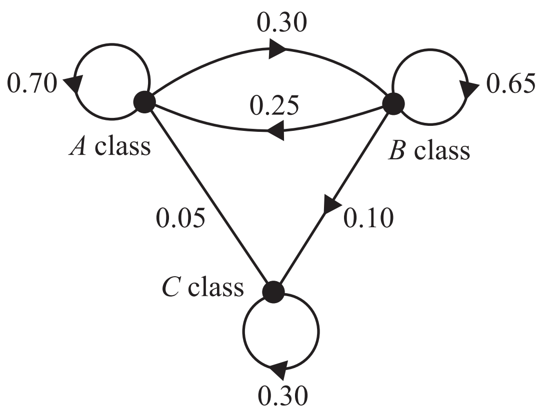
The Whiteoak Theatre Club has 200 members who buy tickets for every concert.

The members can choose seats from three different classes, *A*, *B* or *C*.

For each concert, the choice of seat class for these members can be determined using the transition matrix *T*, shown below.

$$\begin{array}{c} \textit{this concert} \\ \begin{array}{ccc} A & B & C \end{array} \end{array}$$
$$T = \begin{bmatrix} 0.70 & 0.25 & 0.05 \\ 0.30 & 0.65 & 0.65 \\ 0.00 & 0.10 & 0.30 \end{bmatrix} \begin{array}{l} A \\ B \quad \textit{next concert} \\ C \end{array}$$

a. An incomplete transition diagram for matrix *T* is shown below.



Complete the **transition diagram above** by adding all the missing information. 2 marks

(*Answer on the transition diagram above.*)

b. The number of seats in each class chosen by these members for the final concert this year is shown in matrix S_0 below.

$$S_0 = \begin{bmatrix} 16 \\ 96 \\ 88 \end{bmatrix} \begin{array}{l} A \\ B \\ C \end{array}$$

What percentage of these members chose *A* class seats for the final concert this year? 1 mark

For the first concert next year, some members will choose a different seat class from the seat class that they chose for the final concert this year.

c. What percentage of the 200 members are expected to change from *B* class seats at the final concert this year to *A* class seats for the first concert next year? 1 mark

The expected number of these members and their choice of seat class for the *n*th concert next year can be determined using the recurrence relation

$$S_0 = \begin{bmatrix} 16 \\ 96 \\ 88 \end{bmatrix}, \qquad S_{n+1} = TS_n$$

d. Write down the state matrix, S_1, for the expected number of members and their choice of seat class for the first concert next year.

Write your answer correct to one decimal place. 1 mark

e. In the long term, how many members would be expected to buy *B* class seats for a concert? 1 mark

f. It is expected that, beginning from the third concert next year, the Whiteoak Theatre Club will have more members.

Ten new members are expected at every new concert.

For their first concert, new members will not be given a seat choice.

Matrix K_2 contains the expected number of members in each class of seat for the second concert next year.

The expected number of members in each class of seat for the third and fourth concerts next year can be determined by

$$K_3 = TK_2 + B \qquad \text{where} \qquad T = \begin{bmatrix} 0.70 & 0.25 & 0.05 \\ 0.30 & 0.65 & 0.65 \\ 0.00 & 0.10 & 0.30 \end{bmatrix}, \; K_2 = \begin{bmatrix} 61 \\ 116 \\ 23 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix}$$
$$K_4 = TK_3 + B$$

Determine the number of members who are expected to choose *A* class seats for the fourth concert next year.

Round your answer to the nearest whole number. 2 marks

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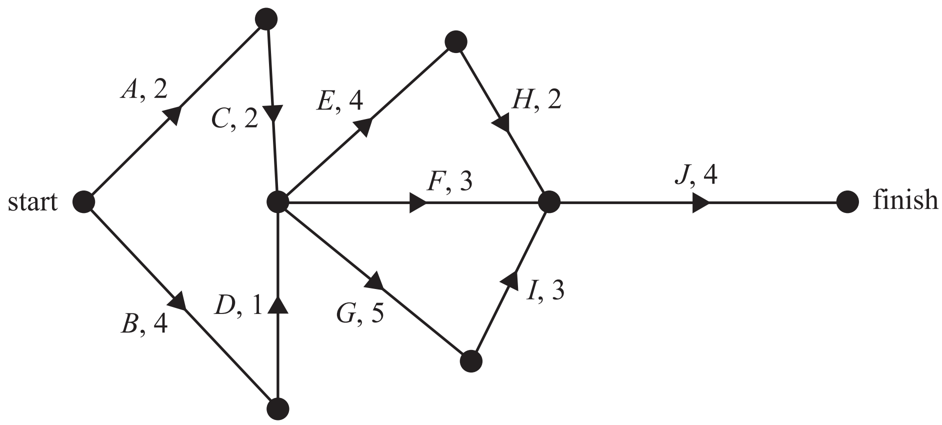
Module 2 – Networks and decision mathematics

Question 1 (5 marks)

Simon is building a new holiday home for his family.

The directed network below shows the 10 activities required for this project and their completion times, in weeks.



a. Write down the two activities that are immediate predecessors of activity *G*. 1 mark

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b. For activity *D*, the earliest starting time and the latest starting time are the same.

What does this tell us about activity *D*? 1 mark

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c. Determine the minimum completion time, in weeks, for this project. 1 mark

---

d. Determine the latest starting time, in weeks, for activity *C*. 1 mark

---

e. Which activity could be delayed for the longest time without affecting the minimum completion time of the project? 1 mark

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Question 2 (4 marks)

The holiday home has four rooms, A, B, C and D.

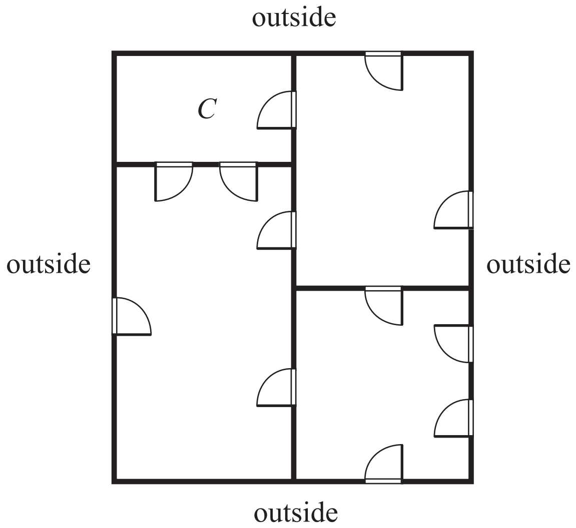
The floor plan, below left, shows these rooms and the outside area.

There are 12 doors, as shown on the floor plan.

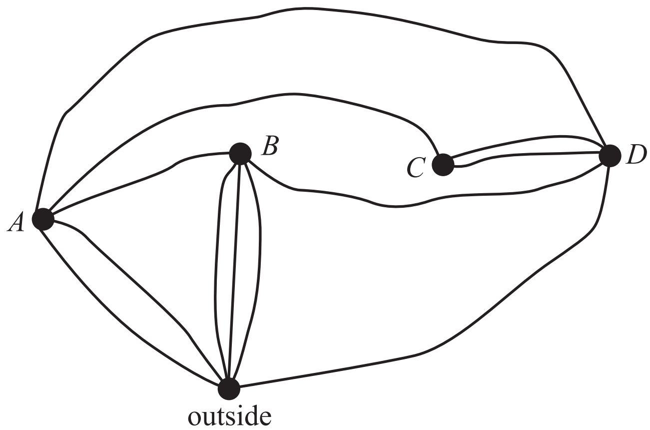
Only room C and the outside are labelled.

A graph is shown to the right of the floor plan. On this graph, vertices represent the rooms and the outside area, and edges represent the doors.

Floor plan



Graph



a. On the floor plan above, room C has already been labelled.

Use the letters A, B and D to label the other three rooms on the **floor plan above**. 1 mark

(*Answer on the floor plan above.*)

b. Simon is in room C and his daughter Zofia is outside.

Simon calls Zofia to see him in room C.

Zofia visits every other room once on her way to room C.

Give the mathematical term that describes Zofia's journey. 1 mark

___

c. Simon tries to find a route that passes through every door once only and finishes back at the starting point.

i. Explain why this is not possible. Refer to the graph in your answer. 1 mark

ii. If two of the doors are locked and only the other doors are considered, then Simon's route will be possible.

Simon locks the door between room *A* and room *C*.

Write down the two rooms that are joined by the other door that must be locked. 1 mark

Question 3 (3 marks)

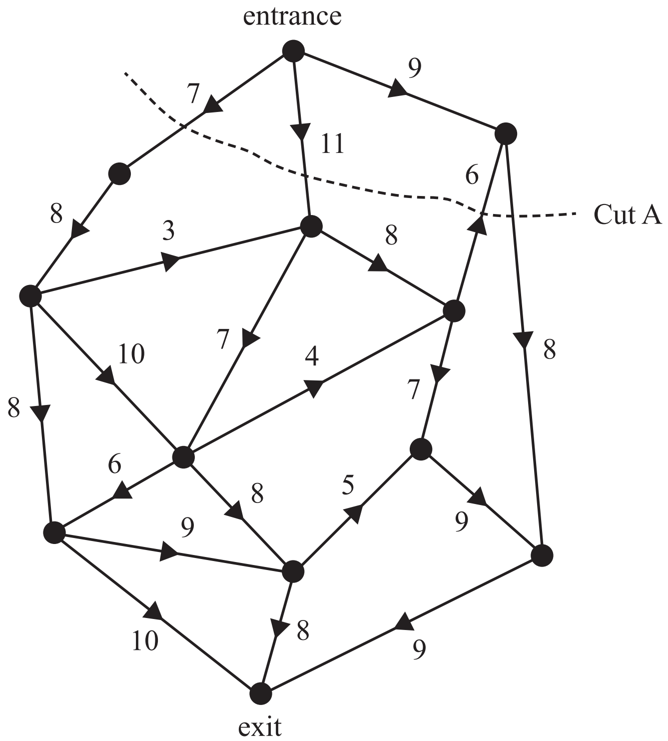
Simon built his holiday home on an estate.

The estate has one-way streets between the entrance and the exit.

There are restrictions on the number of trucks that are allowed to travel along each street per day.

On the directed graph below, the vertices represent the intersections of the one-way streets.

The number on each edge is the maximum number of trucks that are allowed to travel along that street per day.



When considering the possible flow of trucks through this network, many different cuts can be made.

a. Determine the capacity of Cut A, shown above. 1 mark

b. Find the maximum number of trucks that could travel from the entrance to the exit per day. 1 mark

c. A company would like to send one group of trucks from the entrance to the exit.

All trucks in this group must follow each other and travel along the same route.

The trucks in this group will be the only trucks to use these streets on that day.

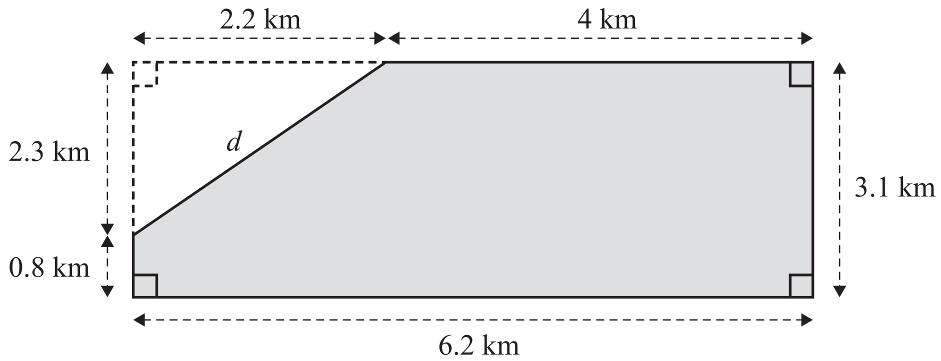
What is the maximum number of trucks that could be in this group? 1 mark

Module 3 – Geometry and measurement

Question 1 (3 marks)

A dairy farm is situated on a large block of land.

The shaded area in the diagram below represents the block of land.



a. Show that the length *d* is 3.2 km, rounded to one decimal place. 1 mark

b. Using $d = 3.2$, calculate the perimeter, in kilometres, of this block of land. 1 mark

c. Calculate the area of this block of land.
Round your answer to the nearest square kilometre. 1 mark

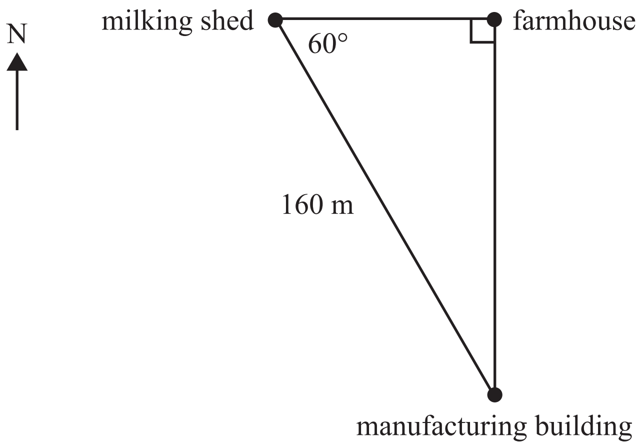
Question 2 (2 marks)

The dairy farm has a farmhouse, a milking shed and a manufacturing building.

The farmhouse is located due east of the milking shed.

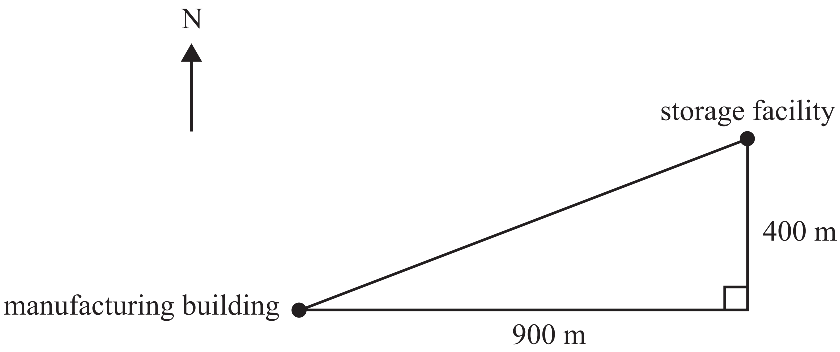
The manufacturing building is located due south of the farmhouse.

The manufacturing building is 160 m from the milking shed, as shown below.



a. How far east of the milking shed is the manufacturing building located? 1 mark

b. A storage facility is located 900 m east and 400 m north of the manufacturing building, as shown below.



What is the bearing of the storage facility from the manufacturing building?

Round your answer to the nearest degree. 1 mark

Question 3 (3 marks)

The dairy farm is located in the town of Milkdale (34° S, 141° E).

a. The cows are milked early in the morning in Milkdale.
One day, the sun rises in another town, Creamville (36° S, 147° E), at 6.42 am.
Assume Milkdale and Creamville are in the same time zone.

At what time will the sun rise in Milkdale on this day? 2 marks

b. Assume that the radius of Earth is 6400 km.

Determine the shortest distance from Milkdale to the South Pole.
Round your answer to the nearest kilometre. 1 mark

Question 4 (4 marks)

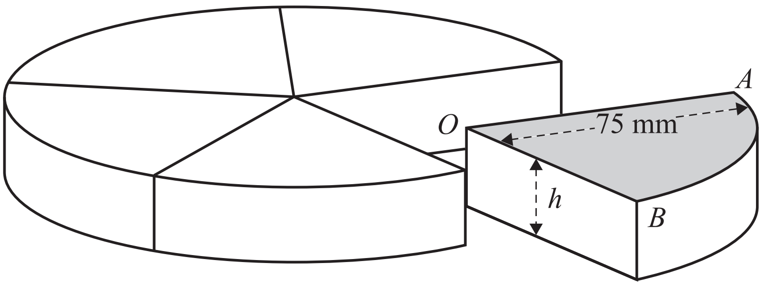
Milk is made into cheese in the manufacturing building.

There are two sizes of cheese, each made in the shape of a cylinder and of equal height. A small cylinder of cheese has a radius of 55 mm and a large cylinder of cheese has a radius of 75 mm.

a. The price of a cylinder of cheese is proportional to its volume.

The price of a small cylinder of cheese is \$12.10

What is the price of a large cylinder of cheese? 2 marks



b. A large cylinder of cheese is cut into five equal pieces and one piece is removed, as shown above.

The area of sector *OAB* (shaded) is 3534.3 mm^2.

The **total** surface area of this piece is 12 200 mm^2.

What is the height, h, of this piece?

Round your answer to the nearest millimetre. 2 marks

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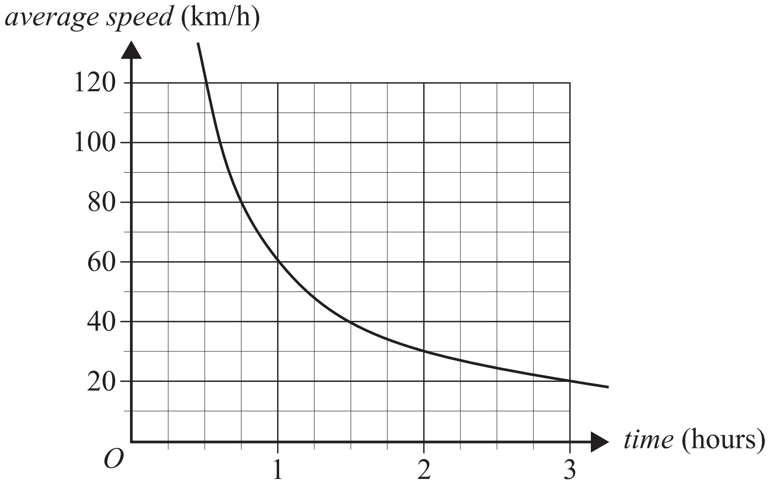
Module 4 – Graphs and relations

Question 1 (4 marks)

Anita often drives to a farmers' market.

The graph below shows the relationship between the *average speed* for the journey (in km/h) and the *time* (in hours) it takes her to reach the farmers' market.



a. One day, it took Anita two hours to reach the farmers' market.

What was her average speed in kilometres per hour? 1 mark

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b. In March, Anita travelled to the farmers' market at an average speed of 80 km/h.

In April, she travelled to the farmers' market at an average speed of 40 km/h due to roadworks.

How much longer did it take Anita to reach the farmers' market in April compared to the time she took in March? 1 mark

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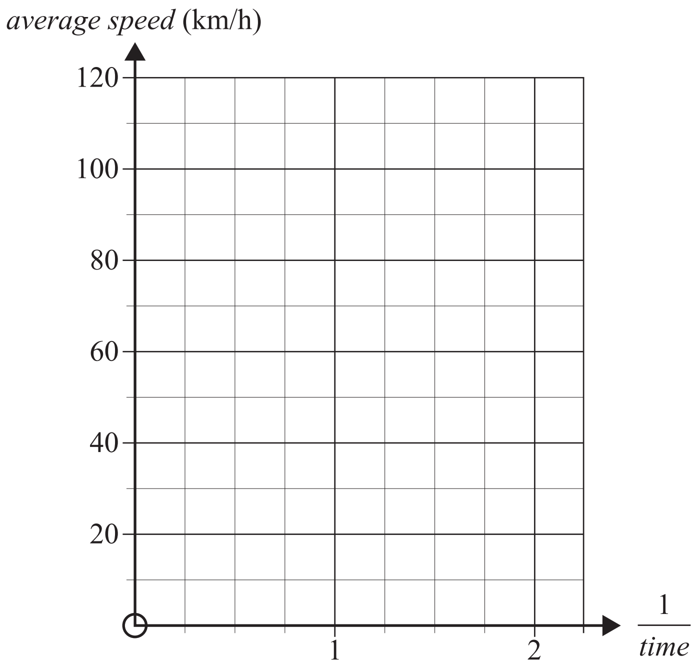
c. The equation for the relationship between *average speed* and *time* has the form

$$average\ speed = \frac{k}{time}$$

i. Find the value of *k*. 1 mark

ii. On the axes provided below, draw a graph of the relationship between *average speed* and $\frac{1}{time}$. 1 mark



Question 2 (4 marks)

Anita sells bottles of tomato juice at the farmers' market.

The *revenue*, in dollars, that she makes from selling n bottles of tomato juice is given by

$$revenue = 6.5n$$

The *cost*, in dollars, of making n bottles of tomato juice is given by

$$cost = 2.5n + 60$$

a. What is the selling price of each bottle of tomato juice? 1 mark

b. How many bottles of tomato juice will Anita need to sell to break even? 1 mark

c. Anita would like to make a profit of \$300 from the sale of 75 bottles of tomato juice.

For this to occur, what would the selling price of each bottle of tomato juice have to be? 2 marks

Question 3 (4 marks)

Anita produces two new flavours of juice, Breakfast Blast and Morning Shine.

Let x be the number of bottles of Breakfast Blast produced each week.

Let y be the number of bottles of Morning Shine produced each week.

Each bottle of Breakfast Blast contains the juice of three apples and one orange.

Each bottle of Morning Shine contains the juice of two apples and two oranges.

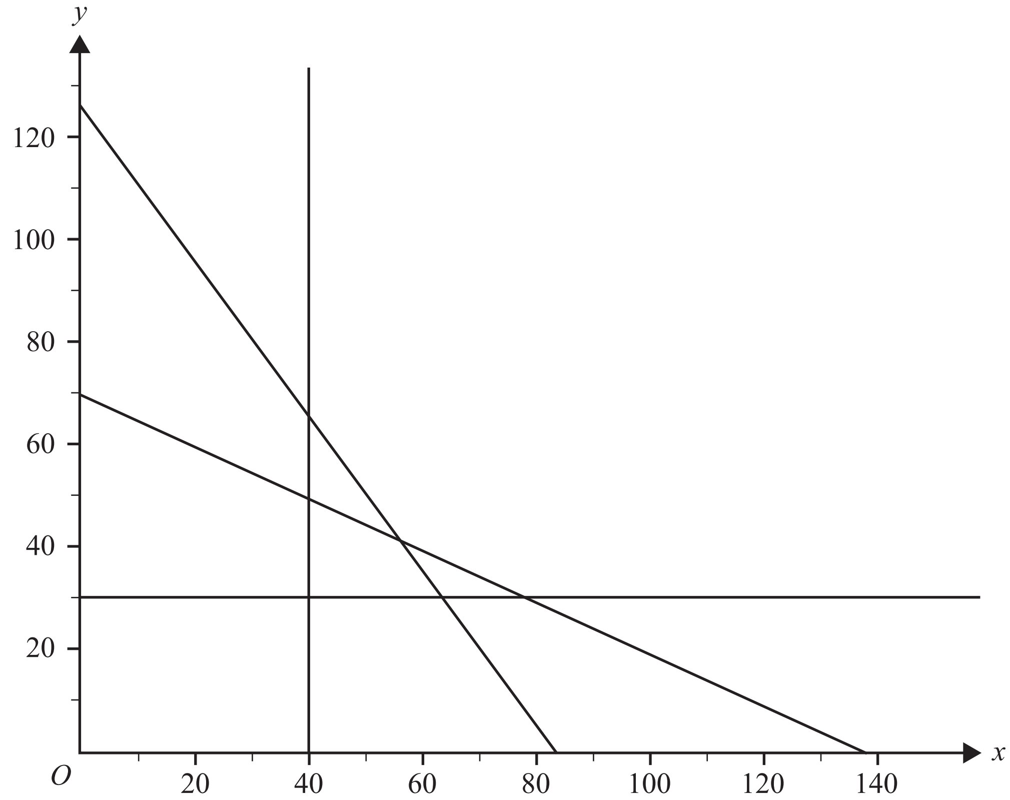
The constraints on the production of juice each week are given by Inequalities 1 to 4.

Inequality 1	$x \geq 40$
Inequality 2	$y \geq 30$
Inequality 3 (apples)	$3x + 2y \leq 250$
Inequality 4 (oranges)	$x + 2y \leq 138$

a. What is the maximum number of oranges available to produce the two flavours of juice each week? 1 mark

b. The graph below shows the lines that represent the boundaries of Inequalities 1 to 4.

On the graph, shade the region that contains the points that satisfy these inequalities. 1 mark



c. Anita makes a profit of $4.80 from every bottle of Breakfast Blast that she produces and $3.20 from every bottle of Morning Shine that she produces.

What is the smallest total number of bottles of the two juices that Anita can produce to make the maximum profit? 2 marks

END OF QUESTION AND ANSWER BOOK

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2017

FURTHER MATHEMATICS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference. A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Further Mathematics formulas

Core – Data analysis

standardised score	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a$, $u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1 – Matrices

determinant of a 2 × 2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2 × 2 matrix	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	S_0 = initial state, $S_{n+1} = T S_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	$v + f = e + 2$

Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc\sin(\theta^\circ)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base × height
volume of a pyramid	$\frac{1}{3}$ × area of base × height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET