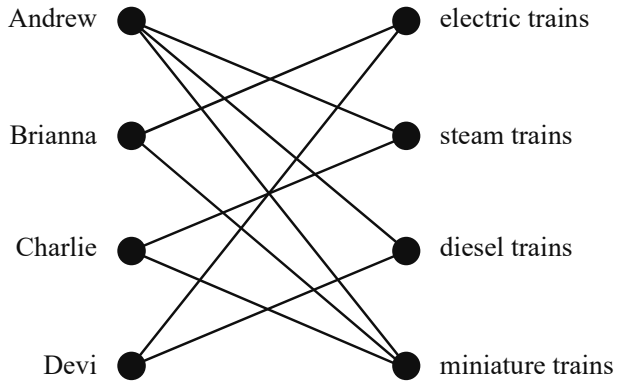
Module 5: Networks and decision mathematics

Question 1 (2 marks)

Four members of a train club, Andrew, Brianna, Charlie and Devi, have joined one or more interest groups for electric, steam, diesel or miniature trains.

The edges of the bipartite graph below show the interest groups that these four train club members have joined.



a. How many of these four members have joined the steam trains interest group? 1 mark

_____

b. Which interest group have both Brianna and Charlie joined? 1 mark

_____

Question 2 (4 marks)

Planning a train club open day involves four tasks.

Table 1 shows the number of hours that each club member would take to complete these tasks.

Table 1

Task	Andrew	Brianna	Charlie	Devi
publicity	13	12	10	10
finances	9	10	11	11
equipment	8	12	11	10
catering	9	10	11	8

The Hungarian algorithm will be used to allocate the tasks to club members so that the total time taken to complete the tasks is minimised.

The first step of the Hungarian algorithm is to subtract the smallest element in each row of Table 1 from each of the elements in that row.

The result of this step is shown in Table 2 below.

a. Complete Table 2 by filling in the missing numbers for Andrew. 1 mark

Table 2

Task	Andrew	Brianna	Charlie	Devi
publicity	3	2	0	0
finances		1	2	2
equipment		4	3	2
catering		2	3	0

After completing Table 2, Andrew decided that an allocation of tasks to minimise the total time taken was not yet possible using the Hungarian algorithm.

b. Explain why Andrew made this decision. 1 mark

Table 3 shows the final result of all steps of the Hungarian algorithm.

Table 3

Task	Andrew	Brianna	Charlie	Devi
publicity	4	2	0	1
finances	0	0	1	2
equipment	0	3	2	2
catering	1	1	2	0

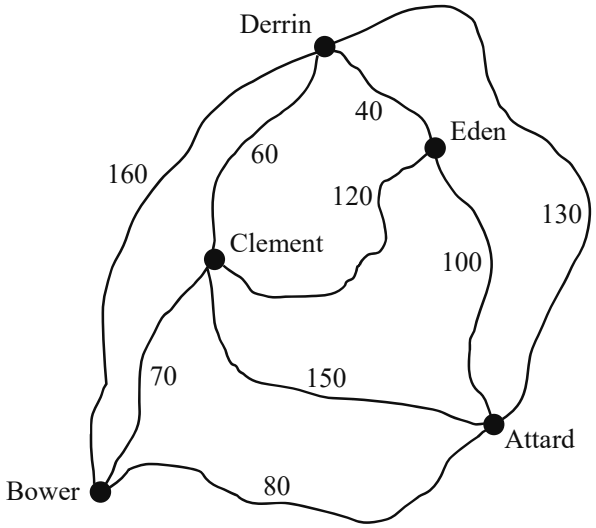
c. **i.** Which task should be allocated to Andrew? 1 mark

ii. How many hours in total are used to plan for the open day? 1 mark

Question 3 (4 marks)

The diagram below shows a network of train lines between five towns: Attard, Bower, Clement, Derrin and Eden.

The numbers indicate the distances, in kilometres, that are travelled by train between connected towns.



Charlie followed an Eulerian path through this network of train lines.

a. **i.** Write down the names of the towns at the start and at the end of Charlie's path. 1 mark

_____

ii. What distance did he travel? 1 mark

_____

Brianna will follow a Hamiltonian path from Bower to Attard.

b. What is the shortest distance that she can travel? 1 mark

_____

The train line between Derrin and Eden will be removed. If one other train line is removed from the network, Andrew would be able to follow an Eulerian circuit through the network of train lines.

c. Which other train line should be removed?

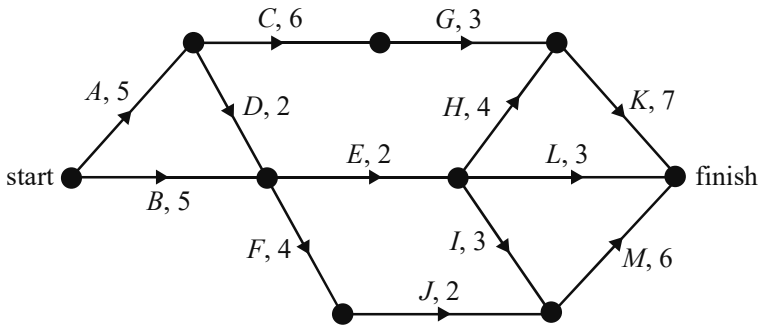
In the boxes below, write down the pair of towns that this train line connects. 1 mark

between ☐ and ☐

Question 4 (5 marks)

To restore a vintage train, 13 activities need to be completed.

The network below shows these 13 activities and their completion times in hours.



a. Determine the earliest starting time of activity *F*. 1 mark

The minimum time in which all 13 activities can be completed is 21 hours.

b. What is the latest starting time of activity *L*? 1 mark

c. What is the float time of activity *J*? 1 mark

Just before they started restoring the train, the members of the club needed to add another activity, *X*, to the project.

Activity *X* will take seven hours to complete.

Activity *X* has no predecessors, but must be completed before activity *G* starts.

d. What is the latest starting time of activity *X* if it is not to increase the minimum completion time of the project? 1 mark

Activity *A* can be crashed by up to four hours at an additional cost of $90 per hour.

This may reduce the minimum completion time for the project, including activity *X*.

e. Determine the least cost of crashing activity *A* to give the greatest reduction in the minimum completion time of the project. 1 mark

END OF MODULE 5
TURN OVER