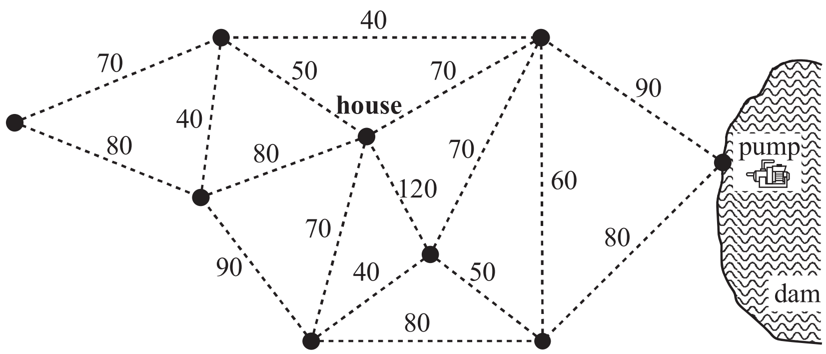
Module 5: Networks and decision mathematics

Question 1

Water will be pumped from a dam to eight locations on a farm.

The pump and the eight locations (including the house) are shown as vertices in the network diagram below.

The numbers on the edges joining the vertices give the shortest distances, in metres, between locations.



a. **i.** Determine the shortest distance between the house and the pump.

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ii. How many vertices on the network diagram have an odd degree?

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iii. The total length of all edges in the network is 1180 metres.
A journey starts and finishes at the house and travels along every edge in the network.
Determine the shortest distance travelled.

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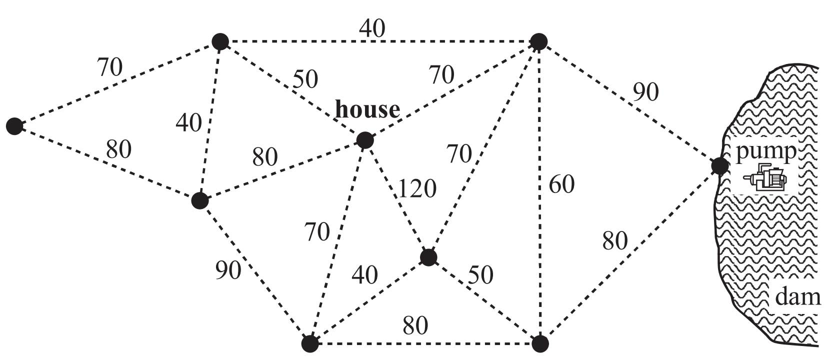
1 + 1 + 1 = 3 marks

The total length of pipe that supplies water from the pump to the eight locations on the farm is a minimum.

This minimum length of pipe is laid along some of the edges in the network.

b. **i.** On the diagram below, **draw** the minimum length of pipe that is needed to supply water to all locations on the farm.



ii. What is the mathematical term that is used to describe this minimum length of pipe in **part i.**?

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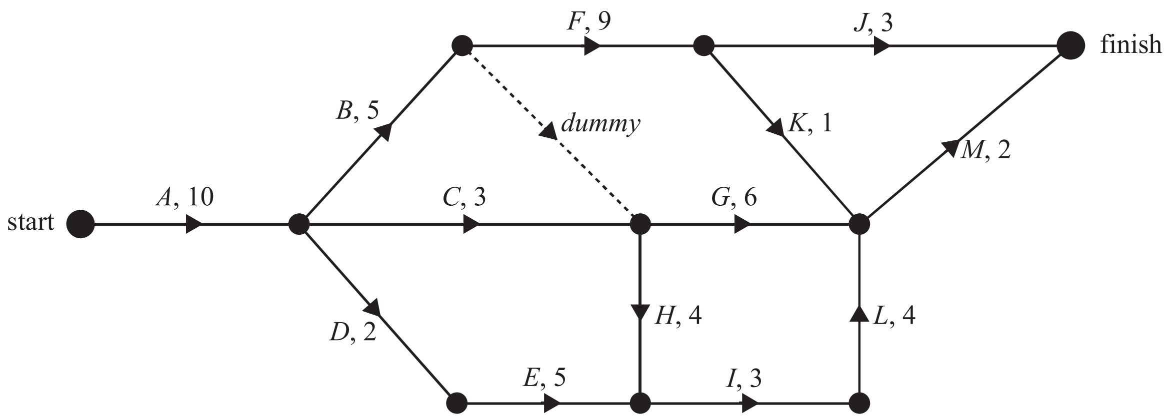
1 + 1 = 2 marks

Module 5: Networks and decision mathematics – continued

Question 2

Thirteen activities must be completed before the produce grown on a farm can be harvested.

The directed network below shows these activities and their completion times in days.



a. Determine the earliest starting time, in days, for activity *E*.

1 mark

b. A *dummy* activity starts at the end of activity *B*.

Explain why this *dummy* activity is used on the network diagram.

1 mark

c. Determine the earliest starting time, in days, for activity *H*.

1 mark

d. In order, list the activities on the critical path.

1 mark

e. Determine the latest starting time, in days, for activity *J*.

1 mark

Question 3

Four tasks, *W*, *X*, *Y* and *Z*, must be completed.

Four workers, Julia, Ken, Lana and Max, will each do one task.

Table 1 shows the time, in minutes, that each person would take to complete each of the four tasks.

Table 1

Task	Julia	Ken	Lana	Max
	Worker			
W	26	21	22	25
X	31	26	21	38
Y	29	26	20	27
Z	38	26	26	35

The tasks will be allocated so that the total time of completing the four tasks is a minimum.

The Hungarian method will be used to find the optimal allocation of tasks.

Step 1 of the Hungarian method is to subtract the minimum entry in each row from each element in the row.

Table 2

Task	Julia	Ken	Lana	Max
	Worker			
W	5	0	1	4
X	10	5	0	
Y	9	6	0	7
Z	12	0	0	9

a. Complete step 1 for task *X* by writing down the number missing from the shaded cell in Table 2.

1 mark

The second step of the Hungarian method ensures that all columns have at least one zero.

The numbers that result from this step are shown in Table 3 below.

Table 3

Task	Julia	Ken	Lana	Max
	Worker			
W	0	0	1	0
X	5	5	0	13
Y	4	6	0	3
Z	7	0	0	5

b. Following the Hungarian method, the smallest number of lines that can be drawn to cover the zeros is shown dashed in Table 3.

These dashed lines indicate that an optimal allocation cannot be made yet.

Give a reason why.

1 mark

c. Complete the steps of the Hungarian method to produce a table from which the optimal allocation of tasks can be made.

Two blank tables have been provided for working if needed.

Worker

Task	**Julia**	**Ken**	**Lana**	**Max**
W				
X				
Y				
Z				

Worker

Task	**Julia**	**Ken**	**Lana**	**Max**
W				
X				
Y				
Z				

1 mark

d. Write the name of the task that each person should do for the optimal allocation of tasks.

Worker	**Task**
Julia	
Ken	
Lana	
Max	

2 marks